

Financial Mathematics

MATH 5870/6870¹
Fall 2021

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¹Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 2. An Introduction to Forwards and Options

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§ 2.1 Forward contracts

§ 2.2 Call options

§ 2.3 Put options

§ 2.4 Options are insurance

§ 2.5 Summary of forward and option positions

§ 2.6 Problems

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Call option : Buyer can walk away.

???? option : Seller can walk away.

Definition 2.3-1 A **put option** gives the owner the right but not the obligation to sell the underlying asset at a predetermined price during a predetermined time period.

Remark 2.3-1 Similar to the call option case, a premium paid by the put buyer at the time the option is purchased is needed in order to compensate the put seller for being in a disadvantage position.

... of put option	someone needs to	premium
seller	buy	has to buy if asked receive
buyer	sell	can walk away pay

Payoff of purchased put = $\max(0, \text{strike price} - \text{spot price at expiration})$

Profit of purchased put = **payoff** of purchased put
– future value of option premium

Payoff of written put = $-\max(0, \text{strike price} - \text{spot price at expiration})$

Profit of written put = **payoff** of written put
+ future value of option premium

Example 2.3-1 S&R Index 6-month European put option

Strike price = \$1,000,

Premium = \$74.20,

6-month risk-free rate = 2%.

Compute both payoff and profit of the **purchased** put option if the index value in six months **\$1,100** (resp. **\$900**).

Solution.

If index value in six months = \$1,100,

$$\begin{aligned}\text{Payoff} &= \max(0, \$1,000 - \$1,100) \\ &= \$0\end{aligned}$$

$$\begin{aligned}\text{Profit} &= \$0 - \$74.20 \times 1.02 \\ &= -\$75.68.\end{aligned}$$

If index value in six months = \$900,

$$\begin{aligned}\text{Payoff} &= \max(0, \$1,000 - \$900) \\ &= \$100\end{aligned}$$

$$\begin{aligned}\text{Profit} &= \$100 - \$74.20 \times 1.02 \\ &= \$24.32.\end{aligned}$$

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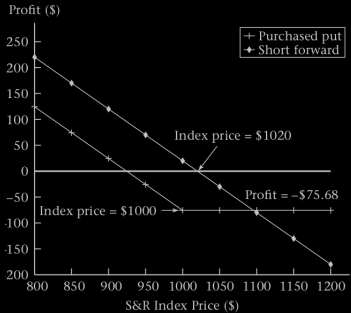
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Example 2.3-2 S&R Index 6-month European put option

Strike price = \$1,000,

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Compute both payoff and profit of the **written** put option if the index value in six months **\$1,100** (resp. **\$900**).

Solution.

If index value in six months = \$1,100,

$$\begin{aligned}\text{Payoff} &= -\max(0, \$1,000 - \$1,100) \\ &= \$0\end{aligned}$$

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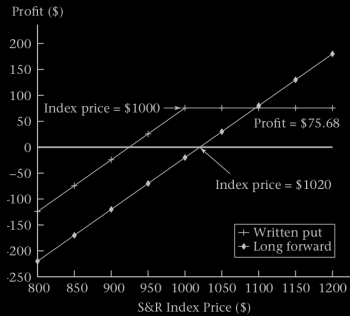
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A **call** option becomes more profitable
when the underlying asset
appreciates in value

A **put** option becomes more profitable
when the underlying asset
depreciates in value

Definition 2.3-2 **Moneyiness** of an option describes whether the option payoff would be positive if the option were exercised immediately.

In particular, one has

Moneyiness	payoff if exercised immediately
In-the-money option	> 0
At-the-money option	$= 0$
Out-of-the money option	< 0