Financial Mathematics

MATH 5870/6870[1]
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[1]Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 2. An Introduction to Forwards and Options

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§ 2.1 Forward contracts

§ 2.2 Call options

§ 2.3 Put options

§ 2.4 Options are insurance

§ 2.5 Summary of forward and option positions

§ 2.6 Problems

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Call option : Buyer can walk away.

???? option : Seller can walk away.

Definition 2.3-1   A put option gives the owner the right but not the obligation to sell the underlying asset at a predetermined price during a predetermined time period.

Remark 2.3-1   Similar to the call option case, a premium paid by the put buyer at the time the option is purchased is needed in order to compensate the put seller for being in a disadvantage position.

... of put option	someone needs to		premium
seller	buy	has to buy if asked	receive
buyer	sell	can walk away	pay

Payoff of purchased put $= \max\left(0, \text{strike price} - \text{spot price at expiration}\right)$

Profit of purchased put $=$ payoff of purchased put
$$- \text{future value of option premium}$$

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Payoff of written put $= -\max\left(0, \text{strike price} - \text{spot price at expiration}\right)$

Profit of written put $=$ payoff of written put
$$+ \text{future value of option premium}$$

Example 2.3-1 S&R Index 6-month European put option

$$\text{Strike price} = \$1,000,$$
$$\text{Premium} = \$74.20,$$
$$\text{6-month risk-free rate} = 2\%.$$

Compute both payoff and profit of the purchased put option if the index value in six months $\$1,100$ (resp. $\$900$).

Solution.

If index value in six months = $\$1,100$,

$$\text{Payoff} = \max(0, \$1,000 - \$1,100)$$
$$= \$0$$
$$\text{Profit} = \$0 - \$74.20 \times 1.02$$
$$= -\$75.68.$$

If index value in six months = $\$900$,

$$\text{Payoff} = \max(0, \$1,000 - \$900)$$
$$= \$100$$
$$\text{Profit} = \$100 - \$74.20 \times 1.02$$
$$= \$24.32.$$

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Example 2.3-2 S&R Index 6-month European put option

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Compute both payoff and profit of the written put option if the index value in six months $1,100 (resp. $900).

Solution.

If index value in six months = $1,100,

$$\text{Payoff} = -\max(0, \$1,000 - \$1,100)$$
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$$\text{Profit} = \$0 + \$74.20 \times 1.02$$
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A call option becomes more profitable
when the underlying asset
appreciates in value

A put option becomes more profitable
when the underlying asset
depreciates in value

Definition 2.3-2 Moneyness of an option describes whether the option payoff would be positive if the option were exercised immediately.

In particular, one has

Moneyness	payoff if exercised immediately
In-the-money option	> 0
At-the-money option	$= 0$
Out-of-the money option	< 0