# Financial Mathematics

MATH 5870/6870[1]
Fall 2021

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Last updated on

September 1, 2021

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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

# Chapter 3. Insurance, Collars, and Other Strategies

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It is always possible

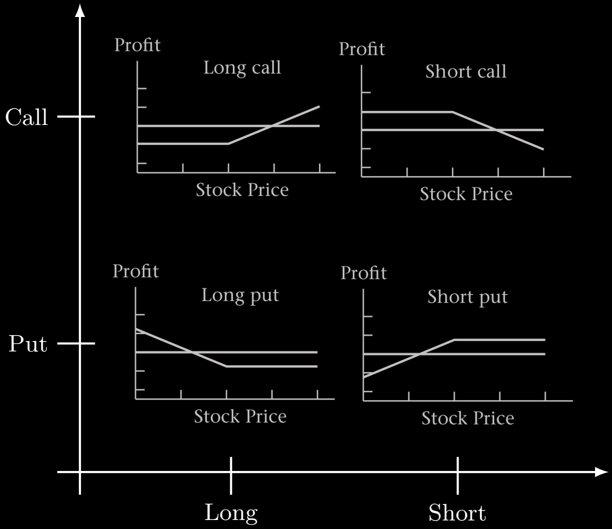
to

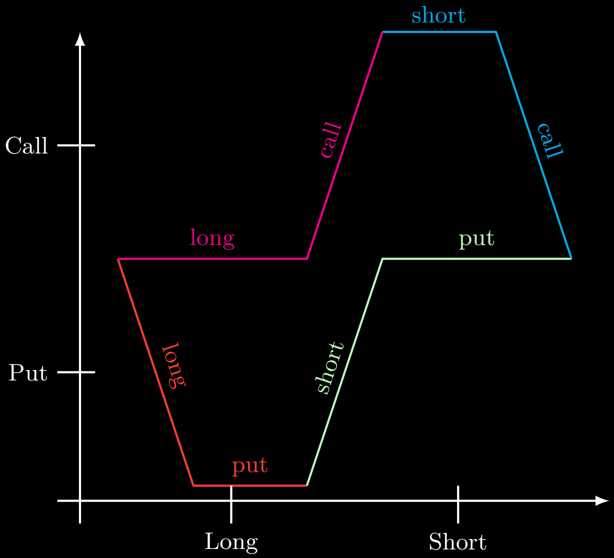
lower the cost of a position

by

reducing its payoff!

By combining two or more options, we find many well-known strategies.




short
call
call
long
put
Call
Put
long
short
put
Long
Short

An option spread is a position consisting of only calls or only puts, in which some options are purchased and some written.

- Bull and bear spreads
- Box spreads
- Ratio spreads
- Collars

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# Example for this section

Black-Scholes option prices

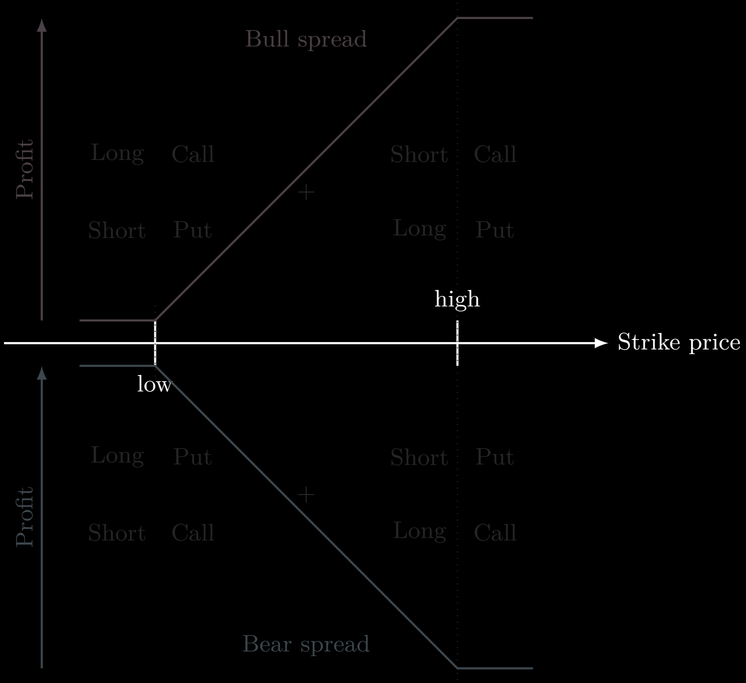
$$\text{Stock price} = \$40$$
$$\text{Volatility} = 30\%$$
$$\text{Effective annual risk-free rate} = 8.33\%$$
$$\text{Dividend yield} = \$0$$
$$\text{Expriation days} = 91 \text{ days}$$

| Strike | Call | Put |
|---|---|---|
| 35 | 6.13 | 0.44 |
| 40 | 2.78 | 1.99 |
| 45 | 0.97 | 5.08 |

# Bull and bear spreads

A position in which you buy a call and sell an otherwise identical call with a higher strike price is an example of a bull spread. Bull spreads can also be constructed using puts.

The opposite of a bull spread is a bear spread.



Bull spread
Profit
Long Call
Short Put
+
Short Call
Long Put
high
Strike price
low
Long Put
Short Call
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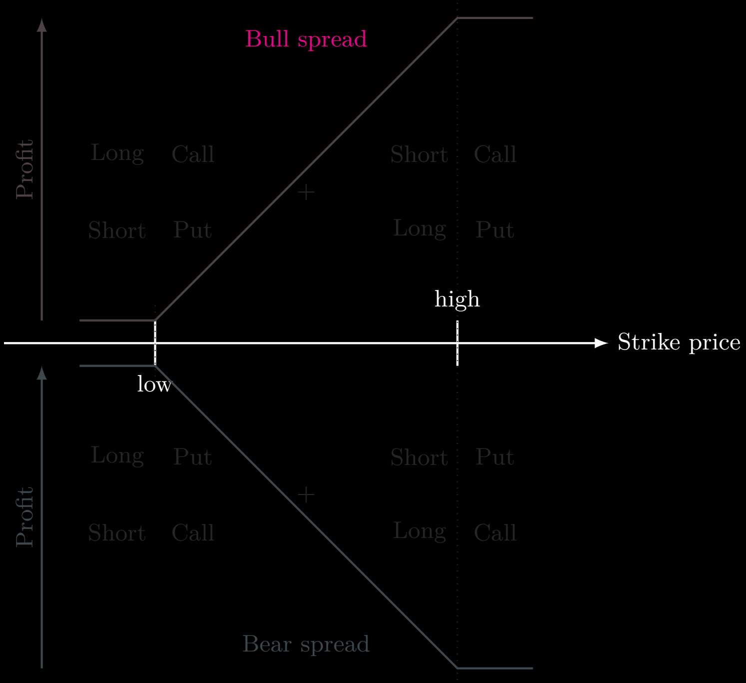
Long Call
Profit
Bear spread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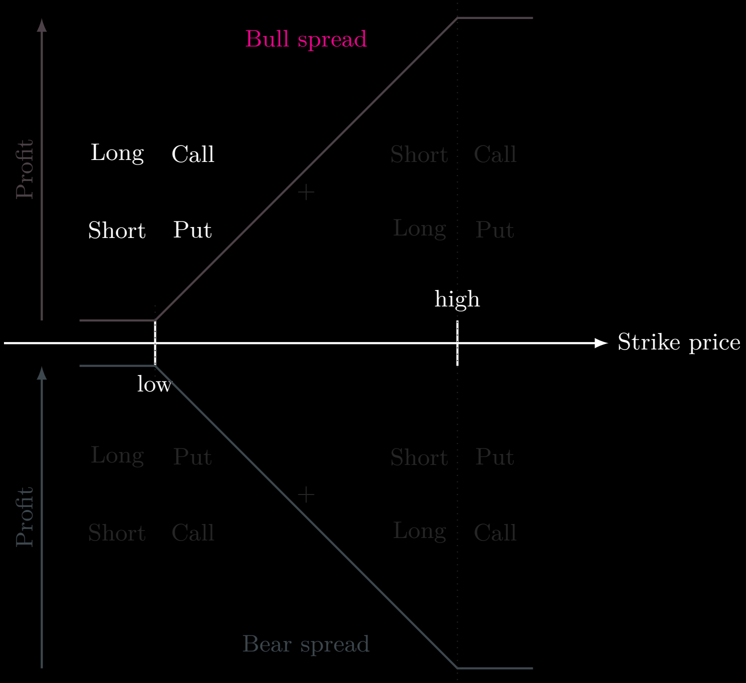
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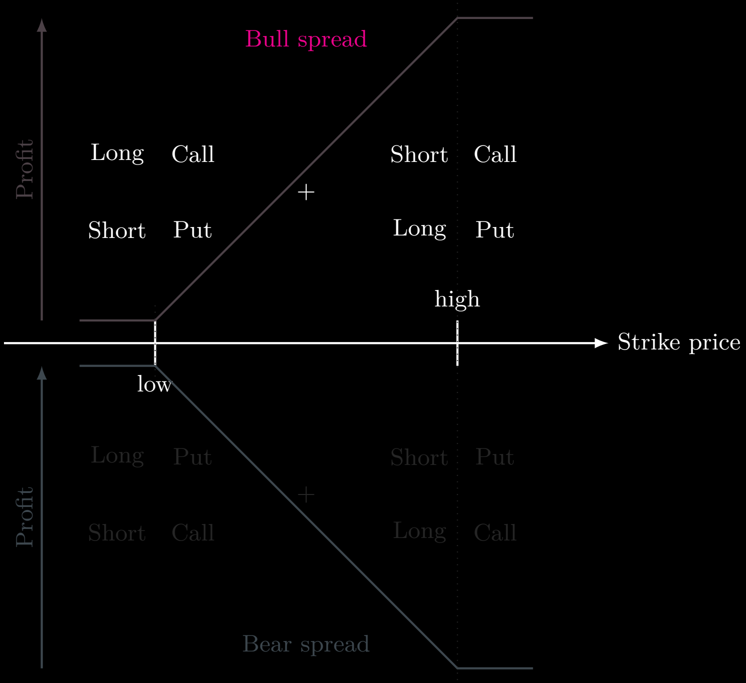

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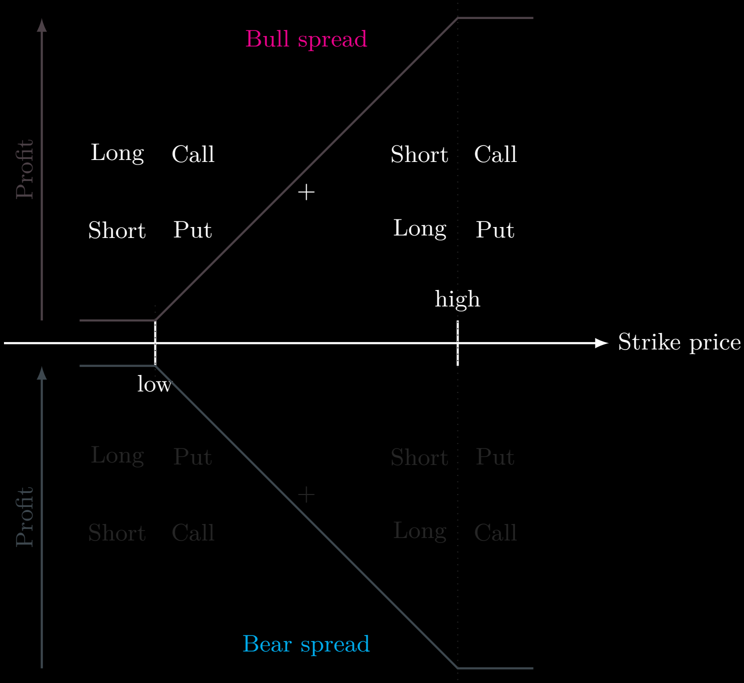
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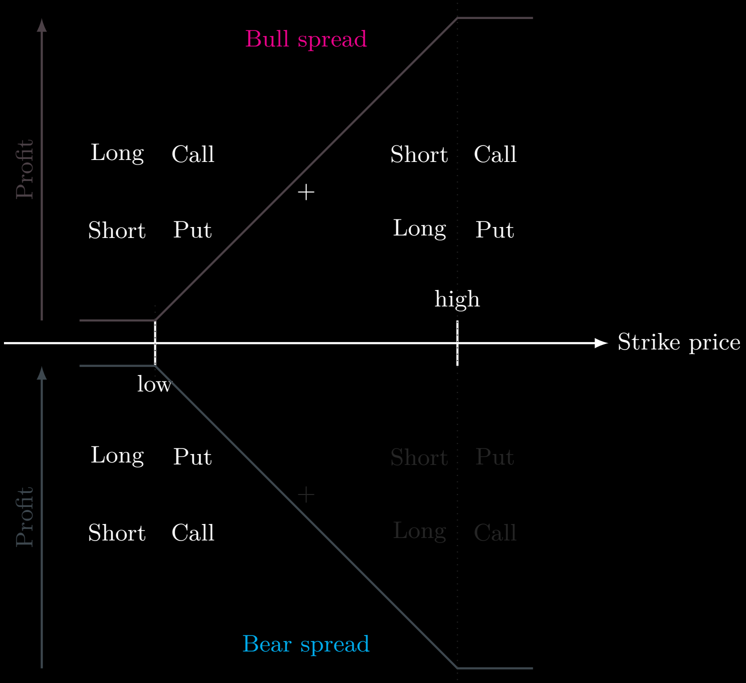
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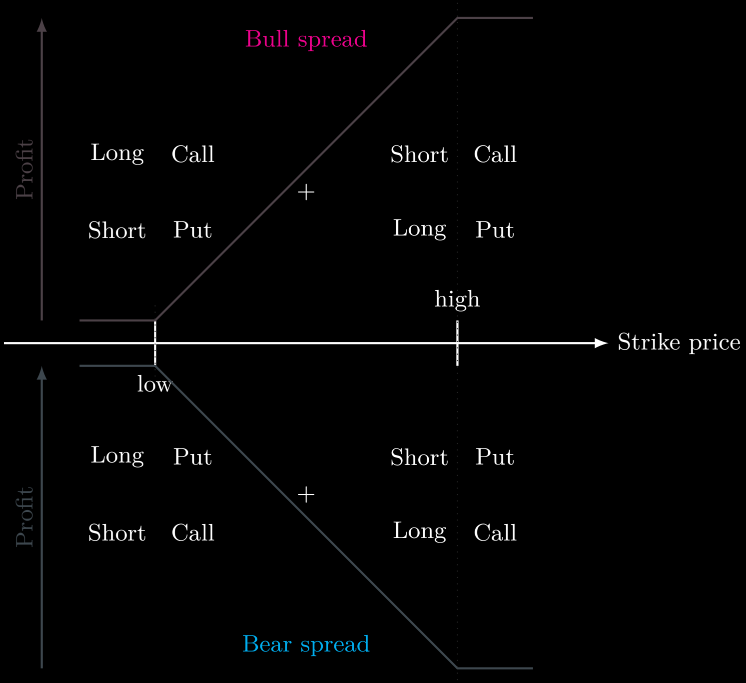
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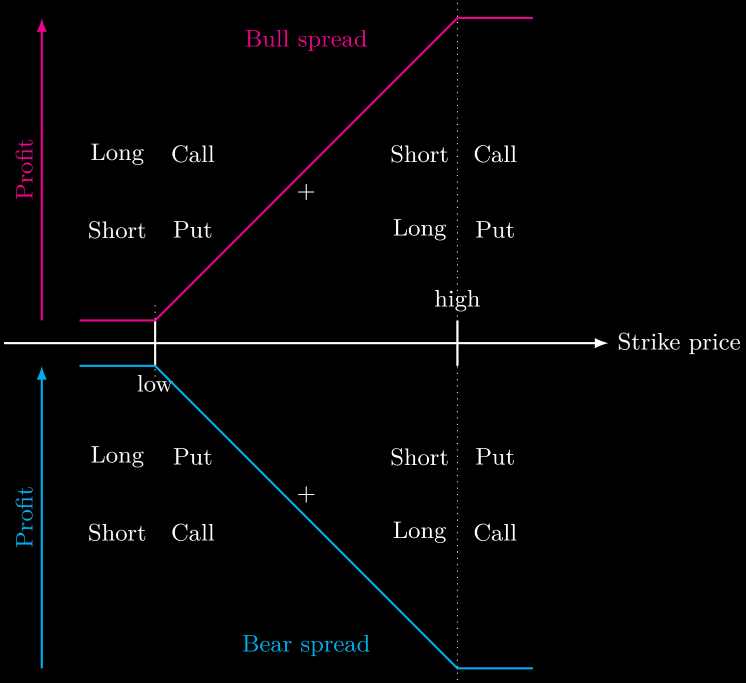
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Example 3.3-1 Draw profit diagram for a 40-45 bull spread, namely, buying a 40-strike call and selling a 45-strike call.

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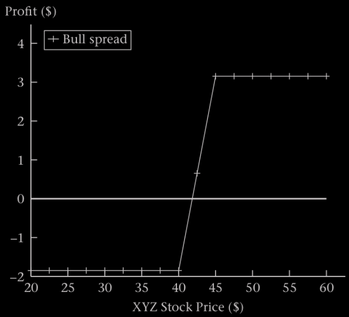


We only need to determine the two levels.

Solution(Continued).

(a) Suppose that the index price is \$ 30 at the expiration:

$$(\$2.78 - \$0.97) \times (1 + 0.0833)^{1/4} = \$1.85.$$

(b) Suppose that the index price is \$50 at the expiration:

$$(\$50 - \$40) - (\$40 - \$45) - \$1.85 = \$3.15.$$

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# Box spreads

A **box spread** is accomplished by using options to create a synthetic long forward at one price and a synthetic short forward at a different price.

This strategy guarantees a cash flow in the future.

Hence, it is an option spread that is purely a means of borrowing or lending money. It is costly but has no stock price risk.

Example 3.3-2 Suppose we simultaneously enter into the following two transactions:

1. Buy a 40-strike call and sell a 40-strike put.
2. Sell a 45-strike call and buy a 45-strike put.

Explain why there is no free lunch. Draw the profit diagram.

Solution. The profit is

$$5+\underbrace{(1.99-2.78)\times(1.0833)^{1/4}}_{\text{Synthetic long forward}}+\underbrace{(0.97-5.08)\times(1.0833)^{1/4}}_{\text{Synthetic short forward}}=\$0.0099851.$$

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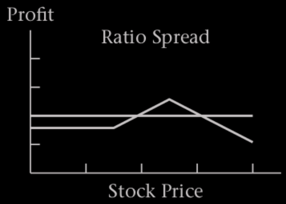
| | | |
|---|---|---|
| Buy a \$40-strike call | Sell a \$40-strike put | Synthetic long forward |
| Sell a \$45-strike call | Buy a \$45-strike put | Synthetic short forward |
| Bull spread | Bear spread | |

# Ratio spreads

A **ratio spread** is constructed by buying m options at one strike and selling n options at a different strike, with all options having the same type (call or put), same time to maturity, and same underlying asset.



Example 3.3-3 (Problem 3.15) Compute profit diagrams for the following ratio spreads:

- a Buy 950-strike call, sell two 1050-strike calls.
- b Buy two 950-strike calls, sell three 1050-strike calls.
- c Consider buying n 950-strike calls and selling m 1050-strike calls so that the premium of the position is zero. Considering your analysis in (a) and (b), what can you say about n/m? What exact ratio gives you a zero premium?

| Strike | Call | Put |
|---|---|---|
| $950 | $120.405 | $51.777 |
| 1000 | 93.809 | 74.201 |
| 1020 | 84.470 | 84.470 |
| 1050 | 71.802 | 101.214 |
| 1107 | 51.873 | 137.167 |

Solution. ... □

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Solution. ... □

# Collars

A **collar** is the purchase of a put option and the sale of a call option with a higher strike price, with both options having the same underlying asset and having the same expiration date.

If the position is reversed, i.e., sale of a put and purchase of a call, the collar is written.

The **collar width** is the difference between the call and put strikes.

Example 3.3-4 Draw the profit diagram for a purchased collar:

selling a 45-strike call + buying a 40-strike put.

Solution. One can easily draw the profit graph. We only need to determine the level when the curve is flat. Hence, suppose the price is \$43. Then the profit is

$$(0.97 - 1.99) \times (1.083)^{1/4} = -\$1.0405.$$

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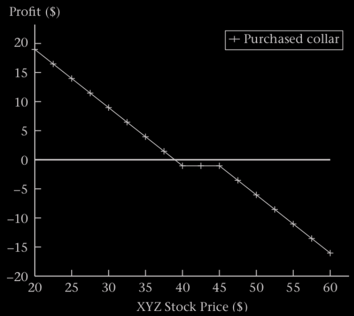


□

It is possible to find strike prices for the put and call such that the two premiums exactly offset one another. This position is called a **zero-cost collar**.

Example 3.3-5 Consider XYZ:

| Strike | Call | Put |
|---|---|---|
| 35 | 6.13 | 0.44 |
| 40 | 2.78 | 1.99 |
| 41.72 | 1.99 | — |
| 45 | 0.97 | 5.08 |

where we need to use Black-Scholes formula to find out the strike price, which is 41.72, when the put premium is \$1.99. This gives a zero-cost collar.