Financial Mathematics

MATH 5870/6870[1]
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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 5. Financial Forwards and Futures

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Three ways to determine the payment for the prepaid forward contracts (no dividend case)

- Pricing the prepaid forward by analogy
- Pricing the prepaid forward by discounted present value
- Pricing the prepaid forward by arbitrage

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Pricing the prepaid forward by analogy

In the absence of dividends, whether you receive physical possession today or at time T is irrelevant: In either case you own the stock, and at time T it will be exactly as if you had owned the stock the whole time. Hence,

$$F_{0,T}^{p} = S_0$$

Pricing the prepaid forward by discounted present value

Let α be the expected return on the stock.

Let $\mathbb{E}_0(S_T)$ be the expected stock price at time T.

Hence,

$$F^p_{0,T} = \underbrace{\mathbb{E}_0(S_T)}_{=S_0 \times e^{\alpha T}} \times e^{-\alpha T} = S_0$$

Pricing the prepaid forward by arbitrage

$$\text{Arbitrage} = \text{Free money}$$

The price of a derivative should be such that

no arbitrage is possible.

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1. If $F_{0,T}^{p} > S_0$: find the arbitrage.
2. If $F_{0,T}^{p} < S_0$: find the arbitrage.

   Hence, $F_{0,T}^{p} = S_0$.

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Pricing prepaid forwards with dividends – Discrete dividends

Suppose a stock is expected to make dividend payments of D_{t_i} at time t_i, $i = 1, \cdots, n$. Then

$$F_{0,T}^{P} = S_0 - \sum_{i=1}^{n} \mathrm{PV}_{0,t_i}\left(D_{t_i}\right),$$

where $\mathrm{PV}_{0,t}(\cdot)$ is the present value at time zero of a time t_j payment.

Example 5.2-1 Suppose XYZ stock costs \$100 today and is expected to pay a \$1.25 quarterly dividend, with the first coming 3 months from today and the last just prior to the delivery of the stock. Suppose the annual continuously compounded risk-free rate is 10%. The quarterly continuously compounded rate is therefore 2.5%. Find a 1-year prepaid forward contract for the stock would cost.

Solution.

$$F^{T}_{0,1} = \$100 - \sum_{i=1}^{4} \$1.25 \times e^{-0.025i} = \$93.30.$$

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Pricing prepaid forwards with dividends – Continuous dividends

Let δ be the compounded dividend yield. Then

$$F_{0,T}^{P} = S_0 e^{-\delta T}$$

Example 5.2-2 Suppose that the index is \$125 and the annualized daily compounded dividend yield is 3%. Find the prepaid forward price at one year.

Solution.

$$F^{p}_{0,1} = \$125e^{-0.03\times 1} = \$121.306.$$

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