Financial Mathematics

MATH 5870/6870[1]
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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 5. Financial Forwards and Futures

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Forward price is the future value of the prepaid forward price:

$$F_{0,T} = \mathrm{FV}\left(F^{p}_{0,T}\right)$$

Example 5.3-1 Continuous dividends

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

- r: risk-free rate.
- δ: the dividend yield.

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Example 5.3-1 Continuous dividends

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- r: risk-free rate.
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$$\text{Forward premium} = \frac{F_{0,T}}{S_0}$$

$$\text{Annualized forward premium} = \frac{1}{T} \ln\left(\frac{F_{0,T}}{S_0}\right)$$

Does the forward price predict the future spot price?

Buying a stock

Compensation for	Earn	Buying a stock
time value of the money	interest	✓
the risk of the stock	risk premium	✓

Entering a forward contract

Compensation for	Earn	Entering a forward contract
time value of the money	interest	✗
the risk of the stock	risk premium	✓

The forward price is the expected future spot price, discounted at the risk premium.

$$F_{0,T} = e^{rT} \times \underbrace{F^{p}_{0,T}}_{=\mathbb{E}_0(S_T)e^{-\alpha T}} = \mathbb{E}_0(S_T)e^{-(\alpha - r)T}$$

Creating a synthetic forward contract

Assuming that the dividends are continuous and paid at the rate δ.

Recall that

Payoff of a long forward position at expiration

$$||$$

$$S_T - F_{0,T}$$

$$||$$

$$S_T - S_0 e^{(r-\delta)T}$$

$$\text{Forward} = \text{Stock} - \text{Zero-coupon bond}$$

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Buy $e^{-\delta T}$ units of the index	$-S_0 e^{-\delta T}$	$+ S_T$
Borrow $S_0 e^{-\delta T}$	$+S_0 e^{-\delta T}$	$- S_0 e^{(r-\delta)T}$
Total	0	$S_T - S_0 e^{(r-\delta)T}$

$$\text{Stock} = \text{Forward} + \text{Zero-coupon bond}$$

Transaction	Cash Flows	
	Time 0	**Time T (expiration)**
Long one forward	0	$S_T - F_{0,T}$
Lend $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_0 e^{(r-\delta)T}$
Total	$-S_0 e^{-\delta T}$	S_T

Zero-coupon bond = Stock − Forward

Transaction	Cash Flows Time 0	Cash Flows Time T (expiration)
Buy $e^{-\delta T}$ units of the index	$-S_0 e^{-\delta T}$	$+S_T$
Short one forward	0	$F_{0,T} - S_T$
Total	$-S_0 e^{-\delta T}$	$F_{0,T}$

Cash-and-carry is a transaction in which one buys the underlying asset and short the offsetting forward contract.

A cash-and-carry has no risk because
You have an obligation to deliver the asset
that you have already owned.

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Buy tailed position in stock, paying $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_T$
Borrow $S_0 e^{-\delta T}$	$+S_0 e^{-\delta T}$	$-S_0 e^{(r-\delta)T}$
Short forward	0	$F_{0,T} - S_T$
Total	0	$F_{0,T} - S_0 e^{(r-\delta)T}$

Cash-and-carry

Transaction	Cash Flows: Time 0	Cash Flows: Time T (expiration)
Buy tailed position in stock, paying $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_T$
Borrow $S_0 e^{-\delta T}$	$+S_0 e^{-\delta T}$	$-S_0 e^{(r-\delta)T}$
Short forward	0	$F_{0,T} - S_T$
Total	0	$F_{0,T} - S_0 e^{(r-\delta)T}$

Arbitrage when $F_{0,T} > S_0 e^{(r-\delta)T}$

Reverse cash-and-carry

Transaction	Cash Flows: Time 0	Cash Flows: Time T (expiration)
Short tailed position in stock, receiving $S_0 e^{-\delta T}$	$+S_0 e^{-\delta T}$	$-S_T$
Lend $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_0 e^{(r-\delta)T}$
Long forward	0	$S_T - F_{0,T}$
Total	0	$S_0 e^{(r-\delta)T} - F_{0,T}$

Arbitrage when $F_{0,T} < S_0 e^{(r-\delta)T}$