Financial Mathematics

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Le Chen

lzc0090@auburn.edu

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Auburn University
Auburn AL

[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 9. Parity and other option relationships

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How does one value the right to back away from a commitment?

TABLE 9.1 IBM option prices, dollars per share, May 6, 2011. The closing price of IBM on that day was $168.89.

		Calls		Puts	
Strike	Expiration	Bid ($)	Ask ($)	Bid ($)	Ask ($)
160	June	10.05	10.15	1.16	1.20
165	June	6.15	6.25	2.26	2.31
170	June	3.20	3.30	4.25	4.35
175	June	1.38	1.43	7.40	7.55
160	October	14.10	14.20	5.70	5.80
165	October	10.85	11.00	7.45	7.60
170	October	8.10	8.20	9.70	9.85
175	October	5.80	5.90	12.40	12.55

Source: Chicago Board Options Exchange.

- What determines the difference between put and call prices at a given strike?
- How would the premiums change if these options were European rather than American?
- It appears that, for a given strike, the October options are more expensive than the June options. Is this necessarily true?
- Do call premiums always decrease as the strike price increases? Do put premiums always increase as the strike price increases?
- Both call and put premiums change by less than the change in the strike price. Does this always happen?

European options

$$C(K,T) - P(K,T) = \mathrm{PV}_{0,T}\left(F_{0,T} - K\right)$$
$$= e^{-rT}\left(F_{0,T} - K\right)$$

Buying a call and selling a put
with the strike both equal to the forward price (i.e., $K = F_{0,T}$)
creates a synthetic forward contract
and hence must have a zero price.

Parity generally fails for American options!

Parity for stocks

$$C(K, T) = P(K, T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT} K$$

Example 9.1-1 Suppose that the price of a non-dividend-paying stock is \$40, the continuously compounded interest rate is 8%, and options have 3 months to expiration. If a 40-strike European call sells for \$2.78, find the price for a 40-strike European put sells.

Solution. Let the price for put be y. Then

$$\$2.78 = y + \$40 - \$40e^{-0.08\times 0.25}$$

Hence,

$$y = \$1.99.$$

$\square$

Why is a call more expensive than a put?

When $S_0 = K$ and $\mathrm{Div} = 0$, then

$$C(K,T) - P(K,T) = K\left(1 - e^{-rT}\right)$$

The difference of a call and put is the time value of money.

Example 9.1-2 Make the same assumptions as in Example 9.1-1, except suppose that the stock pays a \$5 dividend just before expiration. If the price of the European call is \$0.74, what would be the price of the European put?

Solution. Let the price for put be y. Then

$$\$0.74 = y + \left(\$40 - \$5e^{-0.08\times 0.25}\right) - \$40e^{-0.08\times 0.25}$$

Hence,

$$y = \$4.85.$$

$\square$

Synthetic securities

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

- Synthetic stock

$$S_0 = C(K,T) - P(K,T) + \mathrm{PV}_{0,T}(\mathrm{Div}) + e^{-rT}K$$

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

- Synthetic Treasury bill (T-bill)

$$\underbrace{S_0 - C(K,T) + P(K,T)}_{\text{a \textbf{conversion}}} = \mathrm{PV}_{0,T}(\mathrm{Div}) + e^{-rT}K$$

Motivation:

A hedged position that has no risk but requires investment.

T-bills are taxed differently than stocks.

Synthetic securities

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

- Synthetic options

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

$$P(K,T) = C(K,T) - (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) + e^{-rT}K$$