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LINEAR ALGEBRA with Applications

Open Edition

ADAPTABLE | ACCESSIBLE | AFFORDABLE

Adapted for

Emory University

Math 221

Linear Algebra

Sections **1 & 2**

Lectured and adapted by

Le Chen

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le.chen@emory.edu

Course page

http://math.emory.edu/~lchen41/teaching/2021_Spring_Math221

by W. Keith Nicholson

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Supplementary Exercises for Chapter 3

Exercise 3.1 Show that

$$\det\begin{bmatrix} a+px & b+qx & c+rx \\ p+ux & q+vx & r+wx \\ u+ax & v+bx & w+cx \end{bmatrix} = (1+x^3)\det\begin{bmatrix} a & b & c \\ p & q & r \\ u & v & w \end{bmatrix}$$

Exercise 3.2

a. Show that $(A_{ij})^T = (A^T)_{ji}$ for all i, j, and all square matrices A.

b. Use (a) to prove that $\det A^T = \det A$. [*Hint*: Induction on n where A is $n \times n$.]

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b. If A is 1×1, then $A^T = A$. In general, $\det[A_{ij}] = \det\left[(A_{ij})^T\right] = \det\left[(A^T)_{ji}\right]$ by (a) and induction. Write $A^T = \left[a'_{ij}\right]$ where $a'_{ij} = a_{ji}$, and expand $\det A^T$ along column 1.

$$\det A^T = \sum_{j=1}^{n} a'_{j1}(-1)^{j+1}\det[(A^T)_{j1}]$$
$$= \sum_{j=1}^{n} a_{1j}(-1)^{1+j}\det[A_{1j}] = \det A$$

where the last equality is the expansion of $\det A$ along row 1.

Exercise 3.3 Show that $\det\begin{bmatrix} 0 & I_n \\ I_m & 0 \end{bmatrix} = (-1)^{nm}$ for all $n \geq 1$ and $m \geq 1$.

Exercise 3.4 Show that

$$\det\begin{bmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{bmatrix} = (b-a)(c-a)(c-b)(a+b+c)$$

Exercise 3.5 Let $A = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$ be a 2×2 matrix with rows R_1 and R_2. If $\det A = 5$, find $\det B$ where

$$B = \begin{bmatrix} 3R_1 + 2R_3 \\ 2R_1 + 5R_2 \end{bmatrix}$$

Exercise 3.6 Let $A = \begin{bmatrix} 3 & -4 \\ 2 & -3 \end{bmatrix}$ and let $\mathbf{v}_k = A^k\mathbf{v}_0$ for each $k \geq 0$.

a. Show that A has no dominant eigenvalue.

b. Find $\mathbf{v}_k$ if $\mathbf{v}_0$ equals:

i. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

ii. $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

iii. $\begin{bmatrix} x \\ y \end{bmatrix} \neq \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$