

lyryx with Open Texts

LINEAR ALGEBRA with Applications

Open Edition



ADAPTABLE | ACCESSIBLE | AFFORDABLE

Adapted for

Emory University

Math 221

Linear Algebra

Sections 1 & 2

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Course page

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Supplementary Exercises for Chapter 5

Exercise 5.1 In each case either show that the statement is true or give an example showing that it is false. Throughout, \mathbf{x} , \mathbf{y} , \mathbf{z} , \mathbf{x}_1 , \mathbf{x}_2 , ..., \mathbf{x}_n denote vectors in \mathbb{R}^n .

- | | |
|---|---|
| a. If U is a subspace of \mathbb{R}^n and $\mathbf{x} + \mathbf{y}$ is in U , then \mathbf{x} and \mathbf{y} are both in U . | m. If $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ is independent, then $t_1\mathbf{x}_1 + t_2\mathbf{x}_2 + \dots + t_n\mathbf{x}_n = \mathbf{0}$ for some t_i in \mathbb{R} . |
| b. If U is a subspace of \mathbb{R}^n and $r\mathbf{x}$ is in U , then \mathbf{x} is in U . | n. Every set of four non-zero vectors in \mathbb{R}^4 is a basis. |
| c. If U is a nonempty set and $s\mathbf{x} + t\mathbf{y}$ is in U for any s and t whenever \mathbf{x} and \mathbf{y} are in U , then U is a subspace. | o. No basis of \mathbb{R}^3 can contain a vector with a component $\mathbf{0}$. |
| d. If U is a subspace of \mathbb{R}^n and \mathbf{x} is in U , then $-\mathbf{x}$ is in U . | p. \mathbb{R}^3 has a basis of the form $\{\mathbf{x}, \mathbf{x} + \mathbf{y}, \mathbf{y}\}$ where \mathbf{x} and \mathbf{y} are vectors. |
| e. If $\{\mathbf{x}, \mathbf{y}\}$ is independent, then $\{\mathbf{x}, \mathbf{y}, \mathbf{x} + \mathbf{y}\}$ is independent. | q. Every basis of \mathbb{R}^5 contains one column of I_5 . |
| f. If $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is independent, then $\{\mathbf{x}, \mathbf{y}\}$ is independent. | r. Every nonempty subset of a basis of \mathbb{R}^3 is again a basis of \mathbb{R}^3 . |
| g. If $\{\mathbf{x}, \mathbf{y}\}$ is not independent, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is not independent. | s. If $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4\}$ and $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3, \mathbf{y}_4\}$ are bases of \mathbb{R}^4 , then $\{\mathbf{x}_1 + \mathbf{y}_1, \mathbf{x}_2 + \mathbf{y}_2, \mathbf{x}_3 + \mathbf{y}_3, \mathbf{x}_4 + \mathbf{y}_4\}$ is also a basis of \mathbb{R}^4 . |
| h. If all of $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ are nonzero, then $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ is independent. | |
| i. If one of $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ is zero, then $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ is not independent. | b. F |
| j. If $a\mathbf{x} + b\mathbf{y} + c\mathbf{z} = \mathbf{0}$ where a, b , and c are in \mathbb{R} , then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is independent. | d. T |
| k. If $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is independent, then $a\mathbf{x} + b\mathbf{y} + c\mathbf{z} = \mathbf{0}$ for some a, b , and c in \mathbb{R} . | f. T |
| l. If $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ is not independent, then $t_1\mathbf{x}_1 + t_2\mathbf{x}_2 + \dots + t_n\mathbf{x}_n = \mathbf{0}$ for t_i in \mathbb{R} not all zero. | h. F |
| | j. F |
| | l. T |
| | n. F |
| | p. F |
| | r. F |

