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LINEAR ALGEBRA with Applications

Open Edition

Adapted for

Emory University

Math 221

Linear Algebra

Sections **1** & **2**

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Course page

http://math.emory.edu/~lchen41/teaching/2021_Spring_Math221

ADAPTABLE | ACCESSIBLE | AFFORDABLE

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Contents

Supplementary Exercises for Chapter 5

Exercise 5.1 In each case either show that the statement is true or give an example showing that it is false. Throughout, $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$, $\mathbf{x}_1$, $\mathbf{x}_2$, $\dots$, $\mathbf{x}_n$ denote vectors in $\mathbb{R}^n$.

a. If U is a subspace of $\mathbb{R}^n$ and $\mathbf{x}+\mathbf{y}$ is in U, then $\mathbf{x}$ and $\mathbf{y}$ are both in U.

b. If U is a subspace of $\mathbb{R}^n$ and $r\mathbf{x}$ is in U, then $\mathbf{x}$ is in U.

c. If U is a nonempty set and $s\mathbf{x}+t\mathbf{y}$ is in U for any s and t whenever $\mathbf{x}$ and $\mathbf{y}$ are in U, then U is a subspace.

d. If U is a subspace of $\mathbb{R}^n$ and $\mathbf{x}$ is in U, then $-\mathbf{x}$ is in U.

e. If $\{\mathbf{x}, \mathbf{y}\}$ is independent, then $\{\mathbf{x}, \mathbf{y}, \mathbf{x}+\mathbf{y}\}$ is independent.

f. If $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is independent, then $\{\mathbf{x}, \mathbf{y}\}$ is independent.

g. If $\{\mathbf{x}, \mathbf{y}\}$ is not independent, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is not independent.

h. If all of $\mathbf{x}_1$, $\mathbf{x}_2$, $\dots$, $\mathbf{x}_n$ are nonzero, then $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ is independent.

i. If one of $\mathbf{x}_1$, $\mathbf{x}_2$, $\dots$, $\mathbf{x}_n$ is zero, then $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ is not independent.

j. If $a\mathbf{x}+b\mathbf{y}+c\mathbf{z}=\mathbf{0}$ where a, b, and c are in $\mathbb{R}$, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is independent.

k. If $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is independent, then $a\mathbf{x}+b\mathbf{y}+c\mathbf{z}=\mathbf{0}$ for some a, b, and c in $\mathbb{R}$.

l. If $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ is not independent, then $t_1\mathbf{x}_1+t_2\mathbf{x}_2+\cdots+t_n\mathbf{x}_n=\mathbf{0}$ for t_i in $\mathbb{R}$ not all zero.

m. If $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ is independent, then $t_1\mathbf{x}_1+t_2\mathbf{x}_2+\cdots+t_n\mathbf{x}_n=\mathbf{0}$ for some t_i in $\mathbb{R}$.

n. Every set of four non-zero vectors in $\mathbb{R}^4$ is a basis.

o. No basis of $\mathbb{R}^3$ can contain a vector with a component $\mathbf{0}$.

p. $\mathbb{R}^3$ has a basis of the form $\{\mathbf{x}, \mathbf{x}+\mathbf{y}, \mathbf{y}\}$ where $\mathbf{x}$ and $\mathbf{y}$ are vectors.

q. Every basis of $\mathbb{R}^5$ contains one column of I_5.

r. Every nonempty subset of a basis of $\mathbb{R}^3$ is again a basis of $\mathbb{R}^3$.

s. If $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4\}$ and $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3, \mathbf{y}_4\}$ are bases of $\mathbb{R}^4$, then $\{\mathbf{x}_1+\mathbf{y}_1, \mathbf{x}_2+\mathbf{y}_2, \mathbf{x}_3+\mathbf{y}_3, \mathbf{x}_4+\mathbf{y}_4\}$ is also a basis of $\mathbb{R}^4$.

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b. F

d. T

f. T

h. F

j. F

l. T

n. F

p. F

r. F