## Math 221: LINEAR ALGEBRA

## Chapter 1. Systems of Linear Equations §1-6. Application to Chemical Reactions

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Chemical Reactions

## Balancing Chemical Reactions

## Problem

Balance the chemical reaction given below involving tin (Sn), hydrogen (H), and oxygen (0).

$$
\mathrm{xSnO}_{2}+\mathrm{yH}_{2} \rightarrow \mathrm{zSn}+\mathrm{wH}_{2} \mathrm{O}
$$

Solution
Setting up a system of equations in $x, y, z, w$ gives

$$
\begin{aligned}
& \mathrm{Sn}: \mathrm{x}=\mathrm{z} \text { or } \mathrm{x}-\mathrm{z}=0 \\
& \mathrm{O}: \\
& \mathrm{H}: 2 \mathrm{x}=\mathrm{w} \text { or } 2 \mathrm{x}-\mathrm{w}=0 \\
& \mathrm{H}=2 \mathrm{w} \text { or } 2 \mathrm{y}-2 \mathrm{w}=0
\end{aligned}
$$

$$
\text { The augmented matrix is }\left[\begin{array}{rrrr|r}
1 & 0 & -1 & 0 & 0 \\
2 & 0 & 0 & -1 & 0 \\
0 & 2 & 0 & -2 & 0
\end{array}\right]
$$

Solution (continued)
The reduced row-echelon matrix is

$$
\left[\begin{array}{llll|l}
1 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 1 & 0 & -1 & 0 \\
0 & 0 & 1 & -\frac{1}{2} & 0
\end{array}\right]
$$

Letting $\mathrm{w}=\mathrm{t}$, the solution is

$$
\begin{aligned}
\mathrm{x} & =\frac{1}{2} \mathrm{t} \\
\mathrm{y} & =\mathrm{t} \\
\mathrm{z} & =\frac{1}{2} \mathrm{t} \\
\mathrm{w} & =\mathrm{t}
\end{aligned}
$$

We can choose any values for $\mathrm{w}=\mathrm{t}$. Suppose we choose $\mathrm{w}=4$, then $\mathrm{x}=2, \mathrm{y}=4, \mathrm{z}=2$ and the balanced reaction is

$$
2 \mathrm{Sn}_{2}+4 \mathrm{H}_{2} \rightarrow 2 \mathrm{Sn}+4 \mathrm{H}_{2} \mathrm{O}
$$

