Math 1120 Test 01
Dr. Smith, September 20, 2024
Key

Instructions: Show all your work for each problem, if the work is incomplete or incorrect you may not receive full credit for that problem. If you do scratch work, indicate what is scratch work; no credit will be taken off for errors in the scratch work.

1. Find the x values at which the following inequality holds and then sketch the solution of the inequality on the number line:

$$(x+1)(x-2)(x-5) \geq 0.$$

Solution. —— (-1) ++++++++ (2) ——— (5) +++++++++

$$\{x|-1 \leq x \leq 2\} \cup \{x|x \geq 5\}.$$

□

2. Find the equation of the line, in slope intercept form, that has slope $\frac{2}{3}$ and contains the point $(3, -2)$.

Solution.

$$\begin{aligned} \frac{y - -2}{x-3} &= \frac{2}{3} \\ y+2 &= \frac{2}{3}(x-3) \\ y+2 &= \frac{2}{3}x - 2 \\ y &= \frac{2}{3}x - 4. \end{aligned}$$

□

3. Find the equation of the line, in slope intercept form, that contains the points $(2, 5)$ and $(3, 3)$.

Solution.

$$\begin{aligned}
\frac{y-5}{x-2} &= \frac{3-5}{3-2} \\
\frac{y-5}{x-2} &= \frac{-2}{1} = -2 \\
y-5 &= -2(x-2) \\
y-5 &= -2x+4 \\
y &= -2x+9.
\end{aligned}$$

$\square$

4. Find the equation of the line with y-intercept $y=3$ that is perpendicular to the line $y = \frac{3x}{2} + 4$.

Solution.

$$y = \frac{-2}{3}x + 3.$$

$\square$

5. What is the domain of the following function:

$$f(x) = \frac{5x+1}{\sqrt{x^2+4x-5}}.$$

Solution.

$$\begin{aligned}
x^2+4x-5 &> 0 \\
(x+5)(x-1) &> 0
\end{aligned}$$

$$\{x|x < -5\} \text{ and } \{x|x > 1\}.$$

$\square$

6. For the function $f(x) = 5x^2$ calculate the following

$$\frac{f(x+h)-f(x)}{h}.$$

Solution.

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{5(x+h)^2-5x^2}{h}\\
&= \frac{(5x^2+10xh+5h^2)-5x^2}{h}\\
&= \frac{10xh+5h^2}{h}\\
&= 10x+5h \text{ for } h \neq 0.
\end{aligned}$$

□

7. Sketch the function $g(h)$ where

$$g(h) = \frac{(5+h)^2-25}{h}.$$

What it the domain of this function?

Solution.

$$\begin{aligned}
\frac{(5+h)^2-25}{h} &= \frac{(25+10h+h^2)-25}{h}\\
&= \frac{10h+h^2}{h}\\
&= 10+h \text{ for } h \neq 0.
\end{aligned}$$

□

Domain is all $h \neq 0$.

8. For the function $f(x) = x^2$: sketch f and calculate the function obtained by shifting f down 4 units and then 3 units to the right, then sketch the resultant function.

9. Let f be the function $f(x) = |x|$ and let g be the function $g(x) = x - 4$.
   a. calculate $g(f(x))$ and $f(g(x))$.

Solution.

$$\begin{aligned} g(f(x)) &= |x| - 4 \\ f(g(x)) &= |x - 4| \end{aligned}$$

□

b. what are the values $g(f(-2))$ and $f(g(-2))$?

Solution.

$$\begin{aligned} g(f(-2)) &= |-2| - 4 = 2 - 4 = -2 \\ f(g(-2)) &= |-2 - 4| = 6. \end{aligned}$$

□

10. An 18 gallon tank is one-thirds full. If it takes 15 minutes to finish filling it, what is the average rate at which the tank is being filled.

Solution. Two-thirds of 18 is 12 gallons. So 12 gallons divided by 15 minutes is $\frac{12}{15}$ or $\frac{4}{5}$ gal/min. □

11. The following function calculates the volume of a weather balloon as it is being filled with hydrogen where t is in minutes and V is in cubic meters: $V(t) = 10 + 5\sqrt[3]{t}$.

a. what is the volume at $t = 8$ and $t = 125$ minutes.

Solution.

$$\begin{aligned} V(8) &= 10 + 5\sqrt[3]{8} = 10 + 10 = 20 \\ V(125) &= 10 + 5\sqrt[3]{125} = 10 + 5 \cdot 5 = 35 \end{aligned}$$

□

b. what is the average rate at which the volume is increasing between these two times.

Solution.

$$\begin{aligned}\text{ave. rate} &= \frac{35-20}{125-8}\text{ cu. ft./min}\\ &= \frac{15}{117}\text{ cu. ft./min.}\end{aligned}$$

□

12. Argue that the following function is one-to-one:

$$f(x) = \frac{2x+1}{x-3}.$$

Proof. Let

$$\frac{2x+1}{x-3} = \frac{2t+1}{t-3}.$$

Then

$$\begin{aligned}\frac{2x+1}{x-3} &= \frac{2t+1}{t-3}\\ (2x+1)(t-3) &= (2t+1)(x-3)\\ 2xt+t-6x-3 &= 2xt+x-6t-3\\ t-6x &= x-6t\\ 7t &= 7x\\ t &= x.\end{aligned}$$

So the function is one-to-one. □

13. Find the inverse of the following function:

$$f(x) = \frac{3}{2x+5}.$$

Solution.

$$\begin{aligned} x &= \frac{3}{2y+5} \\ x(2y+5) &= 3 \\ 2xy+5x &= 3 \\ 2xy &= 3-5x \\ y &= \frac{3-5x}{2x}. \end{aligned}$$

So the inverse function is

$$g(x) = \frac{3-5x}{2x}.$$

$\square$

Here's an alternate way to do the problem:

Solution.

$$\begin{aligned} x &= \frac{3}{2y+5} \\ \frac{1}{x} &= \frac{2y+5}{3} \\ \frac{3}{x} &= 2y+5 \\ \frac{3}{x}-5 &= 2y \\ \frac{3}{2x}-\frac{5}{2} &= y. \end{aligned}$$

So the inverse function is

$$g(x) = \frac{3}{2x}-\frac{5}{2}.$$

$\square$

14. Sketch the graph of the parabola: $y = x^2 - 6x + 15$. Does it have a high point or a low point? Calculate the x and y coordinates of which ever of the two it has.

Solution.

$$\begin{aligned} y &= x^2 - 6x + 15 \\ &= x^2 - 6x + 9 + 6 \\ &= (x-3)^2 + 6. \end{aligned}$$

So the function has a low point at $(3, 6)$. □

15. Consider the parabola: $y = x^2 - 8x - 12$. Find all the roots of the parabola and sketch the graph.

Solution.

$$\begin{aligned} x^2 - 8x - 12 &= 0 \\ x^2 - 8x + 16 - 16 - 12 &= 0 \\ (x-4)^2 - 28 &= 0 \\ (x-4)^2 &= 28 \\ (x-4) &= \pm\sqrt{28} \\ x &= 4 \pm \sqrt{28}. \end{aligned}$$

You may also use the quadratic formula.

$$\begin{aligned} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} &= \frac{8 \pm \sqrt{8^2 - 4(-12)}}{2} \\ &= 4 \pm \sqrt{28}. \end{aligned}$$

□

Extra Credit: Aunt Polly has a 24-foot fence that she wants whitewashed. If Tom Sawyer can whitewash the fence in 4 hours and his friend Huck Finn can do it in 3 hours, how long will it take them to do it together?

Solution. Tom can whitewash the fence at a rate of 6 fr/hr and Huck can whitewash it at a rate of 8 ft/hr. So together they can whitewash the fence at a rate of 14 ft/hr. So together they can whitewash the fence in

$$\frac{24 \text{ ft}}{14\frac{\text{ft}}{\text{hr}}} = \frac{24}{14} \text{ hrs.}$$

$\square$

The 24 feet is a hint, the problem can be done with specifying the length of the fence.

Solution. Let L denote the length of the fence, then Tom can whitewash the fence at a rate of $L/4$ and Huck can whitewash it at a rate of $L/3$. So together they can whitewash the fence at a rate of $\frac{L}{4} + \frac{L}{3}$ So together they can whitewash the fence in

$$\begin{aligned} \frac{L}{\frac{L}{4}+\frac{L}{3}} &= \frac{L}{L(\frac{1}{4}+\frac{1}{3})} \\ &= \frac{1}{\frac{3+4}{3\cdot 4}} = \frac{12}{7}. \end{aligned}$$

$\square$