Math 1120 Test 02.
Dr. Smith, October 18, 2024

Instructions: Show all your work for each problem, if the work is incomplete or incorrect you may not receive full credit for that problem. If you do scratch work, indicate what is scratch work; no credit will be taken off for errors in the scratch work.

Put your name on each page. **Indicate your answers**, by a box or circle is fine.

Return the test sheet with your work.

1. Use synthetic division to show that -3 is a root of the polynomial $P(x) = x^3 + 2x^2 - 4x - 3$. Find all the roots.

Solution:
$$\begin{array}{r|rrrr} -3 & 1 & 2 & -4 & -3 \\ & & -3 & -3 & -3 \\ \hline & 1 & -1 & -1 & 0 \end{array}$$

The quadratic factor is $x^2 - x - 1$.

The roots are -3 and we use the quadratic formula to get the other two roots:
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}.$$
□

2. Use long division to find the following quotient as a polynomial plus a remainder, where the remainder is a polynomial of degree less than the divisor divided by the divisor:
$$\frac{9x^3 - 6x^2 + 5x - 10}{2x^2 - 8}.$$

Solution:
$$\begin{array}{r|llll} & \frac{9}{2}x & -3 & & \\ \hline 2x^2 - 8 & 9x^3 & -6x^2 & +5x & -10 \\ & \underline{9x^3} & & \underline{-36x} & \\ & & -6x^2 & +41x & -10 \\ & & \underline{-6x^2} & \_ & \underline{+24} \\ & & & +41x & -34 \end{array}$$

Therefore:

$$\frac{9x^3 - 6x^2 + 5x - 10}{2x^2 - 8} \quad = \quad \frac{9}{2}x - 3 + \frac{41x - 34}{2x^2 - 8}.$$

$\square$

3. Given $P(x) = 2x^3 - 3x^2 + 10x - 8$. Use synthetic division to calculate the following
(a.) $P(3)$,
(b.) $P(-2)$.

Solution:

$$\begin{array}{r|rrrr} 3 & 2 & -3 & +10 & -8 \\ & & 6 & 9 & 57 \\ \hline & 2 & 3 & 19 & 49 \end{array}$$

And

$$\begin{array}{r|rrrr} -2 & 2 & -3 & +10 & -8 \\ & & -4 & 14 & -48 \\ \hline & 2 & -7 & 24 & -56 \end{array}$$

So we have:
(a.) $P(3) = 49$,
(b.) $P(-2) = -56$. $\square$

4. (a.) Factor the following polynomial into its linear and quadratic factors:

$$P(x) \quad = \quad x^3 + 1.$$

[Hint: use synthetic division.]

Solution:

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 0 & 1 \\ & & 1 & -1 & 1 \\ \hline & 1 & -1 & 1 & \end{array}$$

So we have:

$$x^3 + 1 \quad = \quad (x+1)(x^2 - x + 1).$$

(b.) Find the real and complex roots of the polynomial.

The roots are -1 and we use the quadratic formula to get the other two roots:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}.$$

□

5. Perform the indicated operations to obtain a result in the form $a + bi$:

$$(a.) \quad (5+7i) - (3-4i) \qquad (b.) \quad (1+3i) \times (-5i).$$

Solution. Part (a):

$$(5+7i) - (3-4i) \;=\; 2 + 11i.$$

Part (b):

$$(1+3i) \times (-5i) \;=\; -5i - 15i^2 = -5i - 15(-1) = 15 - 5i.$$

□

6. Perform the indicated operations to obtain a result in the form $a + bi$:

$$(a.) \quad (i^4 + 2i^3) \times (1-i) \qquad (b.) \quad \frac{3+2i}{1+i}.$$

Solution. Part (a):

$$\begin{aligned} (i^4 + 2i^3) \times (1-i) &= (1-2i) \times (1-i) \\ &= (1-i) - 2i + 2i^2 \\ &= 1 - 3i + 2(-1) = -1 - 3i. \end{aligned}$$

Part (b):

$$\begin{aligned} \frac{3+2i}{1+i} &= \frac{3+2i}{1+i} \times \frac{1-i}{1-i} \\ &= \frac{(3-3i) + (2i - 2i^2)}{1 - i^2} \\ &= \frac{(3-3i) + (2i+2)}{1+1} \\ &= \frac{5-i}{2}. \end{aligned}$$

□

7. (a) Sketch the following function

$$f(x) = \frac{5x-3}{2x-1};$$

(b) find all the straight line asymptotes;
(c) find the domain and range.

Solution. Part (b): Vertical asymptote is $x = \frac{1}{2}$; horizontal asymptotes is $y = \frac{5}{2}$.

Part (c): Domain is $\{x|x \neq \frac{1}{2}\}$; range is $\{y|y \neq \frac{5}{2}\}$. □

8. An investor has placed \$5,000 in a company that promises 6% interest compounded quarterly (four times a year). How much should the investor expect after 3 years?

(a.) Write out the formula;

(b.) indicate the values of the quantities in your formula of part (a.) and then calculate the amount.

Solution. Part (a)

$$A(t) = P\Big(1 + \frac{r}{n}\Big)^{nt}.$$

Part (b)

$$P = \$5,000; r = 0.06; n = 4; t = 3.$$

$$\begin{aligned} A(3) &= \$5,000\Big(1 + \frac{0.06}{4}\Big)^{4\cdot 3} \\ &= \$5978.09. \end{aligned}$$

□

9. Repeat problem 8 but assume that the principle is compounded continuously:

(a.) Write out the formula;

(b.) indicate the values of the quantities in your formula of part (a.) and then calculate the amount.

Solution. Part (a)

$$A(t) = Pe^{rt}.$$

Part (b)

$$P = \$5,000; r = 0.06; t = 3.$$

$$\begin{aligned} A(3) &= \$5,000e^{rt} \\ &= \$5986.09. \end{aligned}$$

□

10. A yeast culture that is well fed doubles it size after 3 days. If the culture started with 1000 cells, how many cells would be in the well fed culture after 9 days?

Solution. After 3 days the culture will have 2000 cells; after 3 more days the culture will have 4000 cells; and after another 3 days the culture will have 8000 cells. So after nine days the culture will have 8000 cells.

□

11. Calculate each of the following (without using your calculator):

$$(a) \quad \log_8(4) \quad (b) \quad \log_{10}(\sqrt{0.001}) \quad (c) \quad \log_5(\sqrt[3]{25})$$

Show your work!

Solution.

$$\begin{aligned} \log_8(4) &= \log_8((8^{\frac{1}{3}})^2) = \log_8(8^{\frac{2}{3}}) = \frac{2}{3}; \\ \log_{10}(\sqrt{0.001}) &= \log_{10}((10^{-3})^{\frac{1}{2}}) = \log_{10}(10^{\frac{-3}{2}}) = \frac{-3}{2}; \\ \log_5(\sqrt[3]{25}) &= \log_5((5^2)^{\frac{1}{3}}) = \log_5(5^{\frac{2}{3}}) = \frac{2}{3}. \end{aligned}$$

□

12. Calculate each of the following (without using your calculator):

$$(a) \quad \ln(e^{2.5}) \quad (b) \quad \ln(\tfrac{e^3}{\sqrt{e}}) \quad (c) \quad \ln\left(\sqrt{\frac{e}{e^3}}\right)$$

Show your work!

Solution.

$$\begin{aligned}
\ln(e^{2.5}) &= 2.5; \\
\ln\left(\frac{e^3}{\sqrt{e}}\right) &= \ln\left(\frac{e^3}{e^{\frac{1}{2}}}\right) = e^{3-\frac{1}{2}} = e^{\frac{5}{2}} = \frac{5}{2}; \\
\ln\left(\sqrt{\frac{e}{e^3}}\right) &= \ln(\sqrt{e^{1-3}}) = \ln(\sqrt{e^{-2}}) = \ln(e^{-1}) = -1.
\end{aligned}$$

□

13. Let L and R be defined as follows

$$\begin{aligned}
L &= \ln(e^a) - \ln(e^b) \\
R &= \ln\left(\frac{e^a}{e^b}\right).
\end{aligned}$$

Calculate L and R and determine if they are equal.

Solution.

$$\begin{aligned}
L &= \ln(e^a) - \ln(e^b) = a - b : \\
R &= \ln\left(\frac{e^a}{e^b}\right) = \ln(e^{a-b}) = a - b.
\end{aligned}$$

□

Extra Credit: [A helpful calculator identity.]
(a.) Argue that

$$b^x = e^{x\ln(b)}.$$

(b.) Use the identity, and your calculator [and indicate what keys you used], to calculate

$$3.1416^{2.72}.$$

Give At least five decimal places.