**Math 1120 Test 03.**

Dr. Smith, November 4, 2024

Instructions: Show all your work for each problem, if the work is incomplete or incorrect you may not receive full credit for that problem. If you do scratch work, indicate what is scratch work; no credit will be taken off for errors in the scratch work. In your calculations give at least 4 significant digits.

Put your name on each page. **Indicate your answers**, by a box or circle is fine.

Return the test sheet with your work.

**Part I 10 points each.** In problems 1 - 5, solve for x.

1. Solve for $x$:

$$\log_5(3x) = 4.$$

*Solution.*

$$\begin{aligned} 3x &= 5^4 \\ x &= 5^4/3 \\ &= 208.333. \end{aligned}$$

□

2. Solve for $x$:

$$\ln(4x+6) - \ln(2x) = \ln(10).$$

*Solution.*

$$\begin{aligned}
\ln(4x+6) - \ln(2x) &= \ln(10) \\
\ln\left(\frac{4x+6}{2x}\right) &= \ln(10) \\
\frac{4x+6}{2x} &= 10 \\
2 + \frac{6}{2x} &= 10 \\
\frac{6}{2x} &= 8 \\
6 &= 16x \\
x &= \frac{6}{16} = \frac{3}{8} = 0.375.
\end{aligned}$$

□

3. Solve for $x$:

$$e^{2x} - 4e^x + 4 = 0.$$

*Solution.* Let $y = e^x$. Then

$$\begin{aligned}
y^2 - 4y + 4 &= 0 \\
(y-2)(y-2) &= 0 \\
y &= 2 \\
e^x &= 2 \\
x &= \ln(2) = .6931.
\end{aligned}$$

□

4. Solve for $x$:

$$e^x - 7 + 12e^{-x} = 0.$$

*Solution.* Let $y = e^x$. Then

$$\begin{aligned} y - 7 + \frac{12}{y} &= 0 \\ y^2 - 7y + 12 &= 0 \\ (y-3)(y-4) &= 0 \\ y = 3 \quad &\text{or} \quad y = 4 \\ e^x = 3 \quad &\text{or} \quad e^x = 4 \\ x = \ln 3 \quad &\text{or} \quad x = \ln 4. \end{aligned}$$

□

5. Solve for $x$:

$$\frac{1}{2} = \frac{20}{10 + 6e^{4x}}$$

*Solution.*

$$\begin{aligned} 10 + 6e^{4x} &= 40 \\ 6e^{4x} &= 30 \\ e^{4x} &= 5 \\ 4x &= \ln 5 \\ x &= \frac{1}{4}\ln 5 = 0.40236. \end{aligned}$$

□

**Part II 20 points each.**

6. The half life of a material is 300 years.

(a.) How much of the material remains after 25 years?

(b.) How long would it take for the material to decay so that only $\frac{1}{3}$ of it remains.

*Solution.* First we find the $k$ in the equation $A(t) = A_0 e^{kt}$

$$
\begin{aligned}
A(t) &= A_0 e^{kt} \\
\frac{1}{2} A_0 &= A_0 e^{k300} \\
\frac{1}{2} &= e^{k300} \\
\ln\left(\frac{1}{2}\right) &= k300 \\
k &= \frac{1}{300} \ln\left(\frac{1}{2}\right).
\end{aligned}
$$

Part (a)

$$
\begin{aligned}
A(t) &= 100 e^{\frac{\ln(0.5)t}{300}} \\
A(25) &= 100 e^{\frac{\ln(0.5)}{12}} \\
&= 94.387\%.
\end{aligned}
$$

Part (b)

$$
\begin{aligned}
\frac{1}{3} &= e^{\frac{\ln(0.5)t}{300}} \\
\frac{300\ln(\frac{1}{3})}{\ln(0.5)} &= t \\
t &= \frac{300\ln(\frac{1}{3})}{\ln(0.5)} = 475.48 years.
\end{aligned}
$$

□

7. A 50 grams sample of a radioactive element decays to 40 grams after 10 years.

(a) What is the equation that gives the amount of the element after $t$ years?

(b) What is the half life of the element?

*Solution.* Part (a)

$$\begin{aligned} A(t) &= A_0 e^{kt} \\ 40 &= 50e^{k10} \\ \frac{4}{5} &= e^{k10} \\ \ln(0.8) &= 10k \\ k &= \ln(0.8)/10 = -0.02231 \\ A(t) &= 50e^{(\ln(0.8)/10)t} = 50e^{(-0.02231)t}. \end{aligned}$$

Part (b)

$$\begin{aligned} 25 &= 50e^{(\ln(0.8)/10)t} \\ 0.5 &= e^{(\ln(0.8)/10)t} \\ \ln(0.5) &= \frac{\ln(0.8)}{10}t \\ t &= \frac{10\ln(0.5)}{\ln(0.8)} = 31.062 \text{ years.} \end{aligned}$$

□

8. Assume that a pod of 150 whales will grow exponentially under a U.N. sanction of whale hunting. Two years after the cessation of whale hunting the population grows from 150 to 180.

(a) How many whales would be expected after 3 years?

(b) How long will the population take to double in size?

*Solution.* First we find the $k$ in the equation $A(t) = A_0e^{kt}$

$$
\begin{aligned}
A(t) &= A_0 e^{kt} \\
180 &= 150 e^{k2} \\
\frac{6}{5} &= e^{k2} \\
\ln(1.2) &= 2k \\
k &= \frac{1}{2}\ln(1.2) = 0.091161.
\end{aligned}
$$

Part (a)

$$
\begin{aligned}
A(t) &= 150 e^{\frac{\ln(1.2)}{2}t} \\
A(3) &= 150 e^{\frac{\ln(1.2)}{2}3} \\
&= 197.18 \Rightarrow 197 \text{ whales.}
\end{aligned}
$$

Part (b)

$$
\begin{aligned}
300 &= 150 e^{\frac{\ln(1.2)}{2}t} \\
2 &= e^{\frac{\ln(1.2)}{2}t} \\
\ln 2 &= \frac{\ln(1.2)}{2}t \\
t &= \frac{2\ln(2)}{\ln(1.2)} = 7.6035 \text{ years.}
\end{aligned}
$$

□

9. A coal mining operation that was polluting a lake was shut down by the EPA. When it was shut down, the pollution in the lake $p(t)$, in grams per cubic meter, was modeled by the following formula:

$$
p(t) = \frac{15}{10 + 2e^{3t}}
$$

where $t$ is measured in years after the mine was shut down.

(a) What was the pollution concentration a year after the mine was shut down?

(b) How long before the pollution is down to 0.1 g/m$^3$?

*Solution.* Part (a)

$$\begin{aligned} p(1) &= \frac{15}{10 + 2e^3} \\ &= 0.29897 g/m^3. \end{aligned}$$

Part (b)

$$\begin{aligned} \frac{1}{10} &= \frac{15}{10 + 2e^{3t}} \\ 10 + 2e^{3t} &= 150 \\ e^{3t} &= 70 \\ 3t &= \ln(70) \\ t &= \frac{1}{3}\ln(70) = 1.4161 \text{ years.} \end{aligned}$$

□