Math 1120 Test 04 Key.
Dr. Smith, November 22, 2024

Instructions: Show all your work for each problem, if the work is incomplete or incorrect you may not receive full credit for that problem. If you do scratch work, indicate what is scratch work; no credit will be taken off for errors in the scratch work. If there are no solutions or ifinitely many, indicate why.

Put your name on each page. **Indicate your answers**, by a box or circle is fine.

Return the test sheet with your work.

Problem 1. Solve the following system of equations:

$$\begin{array}{rcrcl} 3x & - & 2y & = & 13 \\ 4x & + & y & = & 10 \end{array}$$

Solution.

$$\begin{aligned} 4x + y &= 10 \\ y &= 10 - 4x \\ 3x - 2y &= 13 \\ 3x - 2(10 - 4x) &= 13 \\ 3x + 8x - 20 &= 13 \\ 11x &= 33 \\ x &= 3 \\ y &= 10 - 4 \cdot 3 \\ y &= -2. \end{aligned}$$

□

Problem 2. Solve the following system of equations:

$$\begin{array}{rcrcl} 2x & - & 3y & = & 10 \\ -4x & + & 6y & = & 12 \end{array}$$

Solution.

$$\begin{array}{rcrcr} 2x & - & 3y & = & 10 \\ -4x & + & 6y & = & 12 \\ 4x & - & 6y & = & 20 \\ & & 0 & = & 32 \end{array}$$

since the last line is a contradiction, there is no solution. There are numerous other ways of doing this. □

Problem 3. Solve the following system of equations:

$$\begin{array}{rcrcrcr} 6x & + & 5y & - & 4z & = & -10 \\ & & 4y & + & 5z & = & 6 \\ & & & & \frac{z}{4} & = & \frac{1}{2} \end{array}$$

Solution.

$$\begin{aligned} \frac{z}{4} &= \frac{1}{2} \\ z &= 2 \\ 4y + 5z &= 6 \\ 4y + 10 &= 6 \\ 4y &= -4 \\ y &= -1 \\ 6x + 5y - 4z &= -10 \\ 6x - 5 - 8 &= -10 \\ 6x - 13 &= -10 \\ 6x &= 3 \\ x &= \frac{1}{2}. \end{aligned}$$

□

Problem 4. Solve the following system of equations:

$$\begin{array}{rcrcrcr} x & - & 3y & + & 2z & = & 10 \\ 2x & - & 5y & + & 5z & = & 21 \\ x & - & 2y & - & 23z & = & -1 \end{array}$$

Solution.

Row 2 - 2×Row 2 → Row 2,

Row 3 - Row 1 → Row 3:

$$\begin{array}{rcrcrcr} x & - & 3y & + & 2z & = & 10 \\ 0 & & y & + & z & = & 1 \\ 0 & & y & - & 25z & = & -11 \end{array}$$

Row 3 2 Row 2 → Row 3:

$$\begin{array}{rcrcrcr} x & - & 3y & + & 2z & = & 10 \\ 0 & + & y & + & z & = & 1 \\ 0 & + & 0 & - & 26z & = & -12. \end{array}$$

So we have

$$\begin{aligned} z &= \frac{12}{26} = \frac{6}{13} \\ y &= 1 - z = \frac{7}{13} \\ x &= 10 + 3y - 2z = \frac{139}{13} \end{aligned}$$

□

Problem 5. Solve the following system of equations:

$$\begin{array}{rcrcr} x^2 & + & y^2 & = & 9 \\ x^2 & - & y^2 & = & 1 \end{array}$$

Solution.

$$\begin{aligned} x^2 + y^2 &= 9 \\ x^2 &= 1 + y^2 \\ 1 + 2y^2 &= 9 \\ 2y^2 &= 8 \\ y^2 &= 4 \\ y &= \pm 2 \\ x^2 &= 1 + y^2 = 1 + 4 = 5 \\ x &= \pm\sqrt{5} \\ (x, y) &= (\sqrt{5}, 2); (-\sqrt{5}, 2); (\sqrt{5}, -2); (-\sqrt{5}, -2). \end{aligned}$$

□

Problem 6. Solve the following system of equations:

$$\begin{aligned} 2x &+ y^2 &= 5 \\ x &+ 3y &= 7 \end{aligned}$$

Solution.

$$\begin{aligned} x &= 7 - 3y \\ 2(7-3y) + y^2 &= 5 \\ 14 - 6y + y^2 &= 5 \\ y^2 - 6y + 9 &= 0 \\ (y-3)^2 &= 0 \\ y &= 3 \\ x &= 7 - 3y = 7 - 9 = -2. \end{aligned}$$

□

Problem 7. Solve the following system of equations:

$$\begin{aligned} x &- y &= 4 \\ & xy &= 8 \end{aligned}$$

Solution.

$$\begin{aligned} x &= 4 + y \\ xy = (4+y)y &= 8 \\ 4y + y^2 &= 8 \\ y^2 + 4y - 8 &= 0 \\ y &= \frac{-4 \pm \sqrt{16+32}}{2} \\ y &= -2 \pm \sqrt{12} \\ x &= 4 + y = 2 \pm \sqrt{12}. \end{aligned}$$

□

Problem 8. [20 pts.] Consider the following set of inequalities:

$$\begin{array}{rcrcrcl} 3y & - & 3x & - & 6 & < & 0 \\ y & + & 2x & - & 4 & > & 0 \\ y & + & 4x & - & 8 & < & 0 \end{array}$$

(8a.) Sketch the region in the plane that satisfies the equations.
(8b.) Find the vertices (i.e. the intersection points of each pair of lines).

Solution. the vertices are:

$$\Big(\frac{2}{3}, \frac{8}{3}\Big), \Big(\frac{6}{5}, \frac{16}{5}\Big), \Big(2, 0\Big).$$

The region is inside the finite triangular region formed by the three lines and does not include any of the lines. □

Problem 9.[20 pts.] Consider the following set of inequalities:

$$\begin{array}{rcrcl} x^2 & + & y^2 & < & 9 \\ x & - & y & < & 0 \end{array}$$

(9a.) Sketch the region in the plane that satisfies the equations.
(9b.) Find the vertices.

Solution. the vertices are:

$$\Big(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\Big), \Big(-\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\Big).$$

The region is the region inside the circle of radius 3 centered at the origin and above the $y = x$ line. □

Problem 10. [30 pts.] Evaluate the following sums. (Calculate the sum and give exact values.)

$$(a.) \quad \sum_{n=1}^{5} (3n-1) \cdot 4.$$

$$(b.) \quad \sum_{n=3}^{6} n^2.$$

$$(c.) \quad \sum_{n=1}^{4} \frac{1}{2n-1}.$$

Solution.

$$\begin{aligned}
\sum_{n=1}^{5}(3n-1)\cdot 4 &= 2\cdot 4+5\cdot 4+8\cdot 4+11\cdot 4+14\cdot 4 \\
&= 160 \\
\sum_{n=3}^{6} n^2 &= 9+16+25+36 \\
&= 86 \\
\sum_{n=1}^{4}\frac{1}{2n-1} &= \frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7} \\
&= \frac{105+35+21+15}{105}=\frac{176}{105}
\end{aligned}$$

□