MATH 1627 Final Exam Dec. 9, 2021
Dr. Smith.

Show all your work and indicate your reasoning; full credit may not be given for answers without accompanying justifications. Please write your name on all the sheets. You may omit one problem; if all the problems are attempted then the one with the lowest credit will be omitted.

I Problems 1-4. Let R be the finite region in the x-y-plane bounded by the curves $y = \sqrt{x^2 + x}$ and $y = 3x$. Just set up the integral, with correct limits of integration, that calculates the given quantity.

1.) The area of the region R.

2.) The volume of the solid obtained by rotating the region R about the x-axis.

3.) The volume of the solid obtained by rotating the region R about the y-axis.

4.) The centroid (the center of mass) of the region R.

5.) An oil tank is in the shape of a cylinder with circular base having a diameter of 30 ft. and a height of 20 ft. Is is half full of oil with a density of 57 lbs/ft^3. Set up the integral that calculates the amount of work needed to pump out the oil over the top edge of the tank.

II. Evaluate the following integrals.

6.) $$\int (x^3)\sqrt{25 + 9x^2}dx$$

7.) $$\int \frac{\ln x}{x^{\frac{3}{2}}}dx$$

8.) $$\int \frac{x}{(x-2)(x^2+1)}dx.$$

9.) Determine if the following integral exists; if it does, evaluate it:

$$\int_1^\infty x^2 e^{-5x}dx.$$

III. Determine if the following series converge, in each case, indicate what test you are using:

$$10.) \quad \sum_{n=1}^{\infty} \frac{\sqrt{9n+4}}{n^2-10n}$$

$$11.) \quad \sum_{n=1}^{\infty} \frac{n^2-n}{3^n}$$

$$12.) \quad \sum_{n=1}^{\infty} \frac{\ln n}{n^{\frac{3}{2}}}.$$

IV. Find the interval of convergence for the following power series:

$$13.) \quad \sum_{n=1}^{\infty} \frac{(x-3)^n}{2^n\sqrt{n^2+15}}$$

$$14.) \quad \sum_{n=1}^{\infty} \frac{(2n)!\ x^n}{n^n\ n!}.$$

15.) Check the endpoints for # 13.

V. Taylor series.

16.) Find the third degree Taylor polynomial $T_3(x)$ expanded about $a = 2$ for the function $f(x) = (x-1)^{\frac{2}{3}}$.

17.) For the function of problem (16), how good is the estimate $T_3(1.5)$ for the function value $f(1.5)$?

18.) Find the power series expansion for the following function and determine its radius of convergence:

$$f(x) \ = \ \int_0^x te^{-t^3}.$$

19.) For the function of problem (18): How good an estimate of $f(\frac{1}{2})$ is obtained by adding up four non-zero terms of the series?

Extra Credit: Determine the sequential limit of the following sequence and prove from the definition given in class that the sequential limit is what you claim it to be.

$$\left\{2 + \frac{1}{\sqrt{n} - 10}\right\}_{n=1}^{\infty}.$$