MATH1627, Dr. Smith, Test #3 Tuesday Nov. 16, 2021.

Make sure to show all your work. You may not receive full credit if the accompanying work is incomplete or incorrect. If you do scratch work make sure to indicate scratch work - I will not take off points for errors in the scratch work if it is so labeled and will assume that the scratch work is not part of the final answer.

Problem 1. Find the interval of convergence for the following series; check the endpoints and indicate what test you use, and indicate your reasoning, to determine the convergence at the endpoint:

$$\sum_{n=0}^{\infty} \frac{(2x-6)^n}{5^n(4n-13)}.$$

Solution.

$$\frac{1}{2} \leq x < \frac{11}{2}.$$

□

Problem 2. Find the interval of convergence for the following series:

$$\sum_{n=0}^{\infty} \frac{(n!)^2(x-2)^n}{(3n+5)(2n)!}.$$

Solution.

$$-2 < x < 6.$$

□

Problem 3. Find the interval of convergence for the following series:

$$\sum_{n=1}^{\infty} \frac{2 \cdot 6 \cdot 10 \cdot \ldots \cdot (4n-2)x^n}{3^n n!}.$$

Solution.

$$-\frac{3}{4} < x < \frac{3}{4}.$$

□

Problem 4. Find the Taylor series of the following function expanded about $a = 1$:

$$f(x) = (26 + x)^{\frac{4}{3}}.$$

Solution.

$$\begin{aligned} f(x) &= 3^4 + 4(x-1) + \frac{4}{3^4 2!}(x-1)^2 + \\ &+ \sum_{=3}^{\infty} \frac{(-1)^n 4 \cdot 2 \cdot 5 \cdot \ldots \cdot (3n-7)}{3^n 3^{3n-4} n!}(x-1)^n. \end{aligned}$$

$\square$

Problem 5. Express the following integral as a power series:

$$\int_0^{\frac{1}{2}} x \cos(\sqrt{x}) dx.$$

Solution.

$$\int_0^{\frac{1}{2}} x \cos(\sqrt{x}) dx = \sum_{n=0}^{\infty} (-1)^n \frac{(\frac{1}{2})^{n+2}}{(n+2)(2n)!}$$

$\square$

Problem 6. For the series that you found in problem 5, estimate the error that occurs when only the first six non-zero terms are used to approximate the series.

Solution. I accepted either of the following for full credit:

$$\frac{(\frac{1}{2})^8}{(8)(12)!} \quad \text{or} \quad \frac{(\frac{1}{4})^9}{(9)(14)!}$$

$\square$

Problem 7 Extra Credit. Let $f(x)$ be the following function:

$$f(x) = \int_0^x e^{t^2}\, dt.$$

Find the Maclaurin series for f. Find the fiftieth and fifty first derivative of f at zero, $f^{[50]}(0)$ and $f^{[51]}(0)$ respectively; give exact values.

Solution.

$$\begin{aligned} f^{[50]}(0) &= 0 \\ f^{[51]}(0) &= \frac{(51)!}{51(25)!}. \end{aligned}$$

$\square$