Hilbert's Hotel

The mathematician David Hilbert envisioned an infinite hotel with the infinitely many rooms numbered $1, 2, 3, \ldots$. One day the president arrives wanting a room. When the manager checked his register he sees that every room is taken. "Oh my goodness," he says. "There are no vacant rooms." But the bellhop, a young mathematics major, whispers something in the manager's ear which causes the manager to smile. "Your room will be ready in just a few minutes," the manager tells the president. Can you think of something that the bellhop could have suggested to the manager?

The next day the queen of England arrives with her retinue. The queen has infinitely many servants $(E_1, E_2, E_3, \ldots)$. You are the bellhop and the manager turns to you for help. What do you suggest this time?

Now the President and Queen came to the hotel to attend an international United Nations conference. They had arrived early. The manager discovers in the next day's morning newspaper that the U.N. is having a huge conference in the city where infinitely many countries will be attending and each country will bring infinitely many ambassadors. Since Hilbert's hotel is the only hotel in the city, the manager telephones his now favorite bellhop early in the morning to ask how to arrange for rooms for everyone. Thinking that the bellhop may need help keeping track of everyone, he sends the list of expected ambassadors to the bellhop as follows:

$$
\begin{array}{lcccl}
\text{Algeria:} & A_1, & A_2, & A_3, & \ldots \\
\text{Belgium:} & B_1, & B_2, & B_3, & \ldots \\
\text{Canada:} & C_1, & C_2, & C_3, & \ldots \\
\vdots & \vdots & \vdots & \vdots & \ldots
\end{array}
$$

See if you can find a way to help the manager as the bellhop, after some thought, did.