

## The Poincaré Universe.

Main assumptions are equivalent to the following: The Universe consists of the points inside a sphere of radius  $R$  such that the length of a measuring rod at distance  $r$  from the center is given by  $\ell = k(R^2 - r^2)$ . We let  $k = 1$  as our standard unit of measurement. Since the distance  $d(A, B)$  between two objects  $A$  and  $B$  is measured by seeing how many times the standard unit divides into the distance  $d(A, B)$ , it follows that the length of a rod from  $r$  to  $r + \Delta r$  will be approximately  $\frac{\Delta r}{R^2 - r^2}$ . So then the length  $\ell(r_1, r_2)$  of a path along a radial ray from  $r_1$  to  $r_2$  will be:

$$\begin{aligned}\ell(r_1, r_2) &= \int_{r_1}^{r_2} \frac{1}{R^2 - r^2} dr \\ &= \int_{r_1}^{r_2} \left( \frac{\frac{1}{2R}}{R+r} + \frac{\frac{1}{2R}}{R-r} \right) dr \\ &= \frac{1}{2R} (\ln(R+r) - \ln(R-r)) \Big|_{r_1}^{r_2} \\ &= \frac{1}{2R} \left( \ln \left( \frac{R+r_2}{R-r_2} \right) - \ln \left( \frac{R+r_1}{R-r_1} \right) \right) \\ &= \frac{1}{2R} \ln \left( \frac{(R+r_2)(R-r_1)}{(R-r_2)(R+r_1)} \right).\end{aligned}$$

So the radius of the Universe would appear to be:

$$\begin{aligned}\int_0^R \frac{1}{R^2 - r^2} dr &= \frac{1}{2R} \ln \left( \frac{(R+R)R}{(R-R)R} \right) \\ &\rightarrow \infty.\end{aligned}$$