

A problem of points.

The problem: Two rich French noblemen, H... and T..., agreed to play the following gambling game based on the fair toss of a coin. One player selects “heads” and the other selects “tails”. They put a sizable amount of gold in a pot kept by a the local priest (a fair and unbiased person) who also flipped the coin. The first one to reach 5 points wins the pot.

They begin their play, the table surrounded by onlooker, when the game is suddenly interrupted (the revolution had reached the chateau.) Player H had 3 heads and player T had 2 tails when the revolutionaries knocked on the door. The priests has the pot of gold; he comes to you (the parish mathematician) to figure out a fair way to divide the pot based on the likelihood of each player wining. How would you divide the pot?

Solution. The possible outcomes O and probabilities $P(O)$ with the indicated winner for each outcome is listed below.

O	p	$wins$
HH	$\frac{1}{4}$	H
HTH	$\frac{1}{8}$	H
HTH	$\frac{1}{8}$	H

outcome	prob	winner
HH	$\frac{1}{4}$	H
HTH	$\frac{1}{8}$	H
HTTH	$\frac{1}{16}$	H
HTTT	$\frac{1}{16}$	T
THH	$\frac{1}{8}$	H
THTH	$\frac{1}{16}$	H
THTT	$\frac{1}{16}$	T
TTHH	$\frac{1}{16}$	H
TTHT	$\frac{1}{16}$	T
TTT	$\frac{1}{8}$	T

To double check the calculations, I check to make sure that all the prob-

abilities some to 1. Then I calculate the probability of each winning:

$$\begin{aligned}P(\text{T wins}) &= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} \\ &= \frac{5}{16} \\ P(\text{H wins}) &= 1 - P(\text{T wins}) \\ &= \frac{11}{16}.\end{aligned}$$

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