Presentations04 Greek and Arab Mathematics
Solving Polynomial Equations

	Topic/Exercise	Presenter	
1	Assume that the volume of a square based pyramid is 1/3 area of the base times the height. Use the balance method of Archimedes to determine that volume of a right circular cone with base a circle of radius $\boldsymbol{r}$ and height $\boldsymbol{h}$.		
2	Use the results of the above and the balance system that I posted at http://webhome.auburn.edu/~smith01/math3010Sp21/ArchimedesBalanceMethod.pdf to determine the volume of a sphere using the "method" of Archimedes.		
3	Generic presentation: Select a problem from your textbook in the section on Greek mathematics and solve it. Get my okay and confirmation as a presentation topic.		
4	State and solve a Diophantine equation.		
5	Generic presentation: Select a problem from your textbook in the section on Arabian and/or Indian mathematics and solve it. Get my okay and confirmation as a presentation topic. [See if you can find one by Omar Khayyam.]		
6	Show that if *r* is a root of the polynomial $\boldsymbol{x^2+bc+c}$ then $\boldsymbol{x-r}$ is a factor of that polynomial; similarly show that if *r* is a root of the polynomial $\boldsymbol{x^3+bx^2+cx+d}$ then x-r is a factor of that polynomial. [Hint, use long division.]		
7	Show that if $$\boldsymbol{P(x) = x^n + A_{n-1}x^{n-1} + A_{n-2}x^{n-2} + \cdots + A_1 x + A_0}$$ and $$\boldsymbol{P(x) = (x-r_1)(x-r_2)\ldots(x-r_n)}$$ then $\boldsymbol{A_0} = (-1)^{\mathrm{n}}\boldsymbol{r_1}\cdot\boldsymbol{r_2}\cdot\ldots\cdot\boldsymbol{r_n}$ and $\boldsymbol{A_{n-1} = -r_1 - r_2 \ldots - r_n}$. [Hint: do it for the quadratic (n=2) and cubic (n=3) first.]		
8	Problem (translated) from an old text: A man put one pair of rabbits in a certain place entirely surrounded by a wall. How many rabbits can be produced from that pair in a year, if the nature of these rabbits is such that every month each pair bears a new pair which from the second month on becomes productive?		
9	Argue, using #6, that if a polynomial has real coefficients and $\boldsymbol{r_1 = a+bi}$ is a root, then $\boldsymbol{r_2 = a-bi}$ must also be a root. Show that this implies that a polynomial with real coefficients can be factored into quadratic terms with real coefficients. Do the special case of the quadratic and cubic equations first.		
10	Consider the polynomial $$\boldsymbol{P(x) = x^4 + x^3 - 4\,x^2 + 4x - 32}$$ Show that $\boldsymbol{r = 2i}$ is an "imaginary" root of the polynomial. Use that information and the properties that Cardano would have given this root to determine the "real" roots (as the mathematicians of the 16th century would have viewed them.) [Hint: calculate $\boldsymbol{(x-2i)(x+2i)}$ and use long division.] Although the Europeans at this time did not accept negative numbers, some began to work with these "fictitious" numbers to get answers to problems, so feel free to do to.		