The Beginning of Infinitesimal Calculus

|  | Topic/Exercise | Presenter |  |
| :---: | :---: | :---: | :---: |
| 1 | Find the Maclaurin expansion for $e^{x}, \sin x$ and $\cos x$. Then substitute $x \leftarrow i x$ to obtain the identity $e^{i x}=\cos x+i \sin x$; then repeat with $x \leftarrow-i x$ to get another identity; finally solve two equations in two unknowns to get $\sin x$ and $\cos x$ in terms of $e^{i x}$ and $e^{-i x}$. |  |  |
| 2 | Consider the equation $(x+i y)^{2}=0+1 i$. Set up two equations in $x$ and $y$ and find real numbers that satisfy the equations. This will calculate $\sqrt{i}$. |  |  |
| 3 | Prove de Moivre's theorem: $(\cos x+\mathrm{i} \sin x)^{n}=\cos n x+i \sin n x$ <br> Where $i=\sqrt{-1}$. Hint: use induction. |  |  |
| 4 | Look up the hyperbolic trig functions, sinh and cosh; prove the identity: $\cosh ^{2}(x)-\sinh ^{2}(x)=1$ |  |  |
| 5 | Prove that the sum of all the elements in a row of Pascal's triangle is an integral power of 2 . Hint: try induction. |  |  |
| 6 | Use the fact that $\binom{n}{k}=\frac{n!}{k!(n-k)!}$ <br> to obtain the identity $\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$ |  |  |
| 7 | In your textbook select some problems to do from the section on the history of the development of the infinitesimal calculus. |  |  |
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