Presentations08
Calculus and Probability

	Topic/Exercise	Presenter	
1	Present the notes topic: http://webhome.auburn.edu/~smith01/math3010Sp23/EulerConstant.pdf		
2	Present the notes topic: http://webhome.auburn.edu/~smith01/math3010Sp23/GammaFunction.pdf		
3	Present the notes topic: http://webhome.auburn.edu/~smith01/math3010Sp23/SeriesExpansions.pdf		
4	Let $x = \sqrt{1+\sqrt{a}\,i} + \sqrt{1-\sqrt{a}\,i}$. Show that if $a > 0$ then x^2 is positive so that x is the positive square root of a real number. Show that if $a = n^2 - 1$ for n a positive integer then x is a real number which is the square root of an integer.		
5	Descartes discovered the logarithmic spiral $r = e^{a\theta}$ (in modern polar coordinates notation where $a > 0$ and $-\infty < \theta < \infty$.) Let $P_0 = (r_0, \theta_0)$ be a point on the curve with $r_0 > 0$ argue that the curve loops around the origin infinitely many times starting at P_0 and use the following arclength formula (in polar coordinates) to show that the length of this portion of the curve is finite: $$\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta.$$		
6	Jacob Bernoulli used the following technique to find the sum $\sum_{i=1}^{n} \frac{1}{i(i+1)}$ He considered the sums: $$S = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n+1} + 0$$ $$S \;=\; 0 + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} + \frac{1}{n+1}$$ and subtracted. Use his technique to obtain the desired sum. [Hint: partial fractions may be helpful.]		
7	Use Leibnitz's differential technique to derive the arclength integral formula. [Hint: if dx^2 and dy^2 are the horizontal and vertical sides of a right triangle, if dh denotes the hypotenuse of the triangle, then $dx^2 + dy^2 = dh^2$.]		
8	Hyperbolic functions. The hyperbolic sinh and cosh are defined as follows: $$\sinh x = \frac{1}{2}(e^x - e^{-x}) \quad \cosh x = \frac{1}{2}(e^x + e^{-x}).$$ And: $$\tanh x = \frac{\sinh x}{\cosh x}, \qquad \operatorname{sech} x = \frac{1}{\cosh x}.$$ Show the following hyperbolic trig-like identities: 1. $\cosh^2 x - \sinh^2 x = 1$ 2. $\tanh^2 x + \operatorname{sech}^2 x = 1$ and 3. using the regular trig functions as a hint, derive the sum and difference formulas: $\sinh(\alpha \pm \beta) = \cdots, \cosh(\alpha \pm \beta) = \cdots$.		
9	Show that the following functions,		

	$$\sinh \omega t, \cosh \omega t, \sin(i\omega t), \cos(i\omega t)$$ all satisfy the following differential equation: $$\frac{dy}{dt} = \omega^2 y.$$		
10	In 1777 Georges-Louis Leclerc, Comte de Buffon posed the following problem (now called Buffon's needle problem): A floor is composed of parallel strips of wood all of width ℓ and a needle (of negligible width) is dropped randomly onto the floor. What is the probability that the needle, when it lands, intersects an edge between two floorboards? Assume the length of the needle is also ℓ.		