Applications of the Modified Socratic Method
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A Short Explanatory Essay

I teach using a pedagogical technique often called the Modified Socratic Method or, in its more modern incarnation, Inquiry Based Learning (IBL). There is research based evidence that many variations of the technique work. The original technique, as presented in Plato's dialogues, is a question and answer exchange between teacher (Socrates in Plato's dialogues) and student. Socrates, as representing the teacher, would lead the student to learn, or discover, something that he wanted the student to understand by asking a sequence of guiding questions that would lead the student to "figure" out the "solution". Plato presents an explanatory application of the technique in the *Meno* where Plato has Socrates lead a young boy, a servant of Socrates' friend Meno, to figure out a version of the Pythagorean Theorem. Now, when Socrates asks the student (the young boy) what is the length of the diagonal of a square of one unit on each side, the student DID NOT whip out his iPhone, google it (or asked ChatGPT) and tell Socrates the answer! Note that had he done so, the student would likely still not know why the answer is what it is – which was Socrates' purpose in the first place: to get the student to figure out how to obtain the solution.

In the *Meno*, and in fact in most of Plato's dialogues, the questions and answers follow each other in the space of a conversation – and I will sometimes do that myself in trying to get a point across in class. So the amount of time that the student has to come up with an "answer" is very short in this classical version of the technique. In my more modern version of the technique, the amount of time available for students to figure out their answers is much greater. I will pose some questions to the class – typically in the form of exercises and questions about the material under consideration – and you will have at least a couple of days, from one class to the next, to think about the answers. For the most part, you will be able to answer some "questions" after thinking about them for a day or two; but there will be questions that will take more than two days to answer and in fact there will be some that will take a week or more and even some that may take weeks.

My manifestation of the IBL technique for History of Mathematics 3010 is to ask students to work through exercises that I posed in class or that are from your textbook. Each student is expected to work through and prepare a solution to the exercises for presentation in class. (**Yes, I expect all the students to work through all the exercises as part of their class preparatopm**.) I will select a student from my (randomized) roll to present her/his solution to an exercise. The rest of the class is expected to critique the presentation for correctness and understanding – I expect, and encourage, students to ask questions. Sometimes the problem being presented will be one to which you did not figure out the solution – in that case you should see what approach was used to construct a solution that was successful so that you can apply the technique yourself to other problems. If the problem is of particular interest and few students have figured out the solution, I may give the rest of the class another couple of days to work on it before asking for it to be presented – sometimes providing hints.

There are two challenges to the modernization of my version of the “Modified” Socratic Method: What to do about looking things up on the internet and what to do about study groups. Here are my expectations:
About the internet (and the more primitive information retrieval systems like libraries): I expect you to attempt to try to find the solutions to exercises without any outside assistance. Since thie is a “History” class – you are allowed to use the internet (in particular Wikipedia and ChatGPT) for Identifications and historical terms, but I expect that you will not look-up solutions to problems unless you have been asked to explain a historical technique. Now, and I’m aware that there is a catch – I’m aware that many of you may likely already done some exploratory research before this course, possibly to satisfy your own curiosity. If you have already seen the solution of a exercise that I ask you to present then you should indicate as much – priority for presentation will be for the students who have figured out the solutions on their own without such assistance. Don’t worry, you’ll have ample opportunities to make presentations of other problems. Not infrequently a student will have seen or read about a concept presented in class and may have seen some “facts” about these concepts; the student may want to use those “facts” learned from the internet (or wherever). In these cases, let me know what you want to use and I’ll formulate a sequence of lemmas that lead to their proofs so that they may be used for proofs of other class theorems.

About study groups: Once an exercise has been presented or theorem proven on the blackboard, then it’s fine to discuss the solutions. In fact I don’t mind encouraging you to get together with some friends (from the class – not outside help) to go over the solutions to problems so that you understand the solutions. Eventually similar problems will appear on tests and it’s a good idea to practice your solutions with a study group. If you have a vibrant study group then I anticipate that the following will happen (a short dialogue, student A is speaking with student B):

A: Say, did you figure out the exercise about the Babylonian roots?

B. No! All I could do was prove that the cube root works.

A. Well I couldn’t prove that an of them converge, but I know Newton’s method.

So in a case like this my approach is to do what professional mathematicians do. When I ask if anyone has the exercise, one of students can indicate that they have a collaborative solution. Then using some randomizing selection devise (e.g. a coin flip) I’ll ask one of the two (or in some cases more) students to present the group solution – the other student is encouraged to add details. And credit it given to each student for their contribution to the solution. A similar scenario is the following:

C. What theorem are you working on?

D. Theorem 125. 5.

C. I was working on that one too. I can do it if the space X is first countable.

D. Well, I can prove that!

My approach is the same: I'd ask student D to prove his theorem first and then ask student C to use student D's result to prove the theorem.

A cautionary tale. A student in a class that I taught many semesters ago presented some very nice proofs of some of the theorems stated in the notes. The proofs were what I'd call "textbook" perfect. While the student had a good presentation grade, the student did not pass the final exam and had a failing test average, so the student did not pass the course. I discourage a dependency on outside help because such "help" may not lead to a good understanding of the underlying mathematics. (Note: some students have on their own come up with "textbook perfect" proofs. I'm always delighted when this happens. – and some of my own students who've discovered some classic proofs as well as their own Rube-Goldberg proofs have gone on to obtain Ph.D.'s.)

My recommended studying procedure: There will be a number of exercises that are almost "obvious": work through those carefully and write up the solutions (and keep them in your notebooks). This will help you understand the underlying structure. Spend at least a day or two working on and thinking about less obvious problems. When you arrive at a solution and another student is presenting the problem, make sure that their logic is correct and that you can see that their solution is correct. Ask questions if you are not sure. When a student presents the solution to a problem that you have not solved, watch the solution, carefully take notes and then when you get home try to solve the problem without using your notes. Then, again, add your solution into your notebook. Use your notes as hints if you get stuck. If you still do not see how to do the problem, in spite of your notes, then this is a good question to bring up to a study group. If the study group doesn't help you, then ask in class (actually you may want to ask in class in any case - if you don't understand the solution the probability is 99.99% that someone else in the class also doesn't understand the solution.)

By the way – my notes are always an "in progress" project and so you should expect typos and errors. Even mathematical errors: I have on occasion forgotten to add some hypothesis to a theorem or exercise. (A caveat: sometimes I will do this deliberately to keep students on their toes.) If you can't solve a problem, but you can solve it with the addition of some extra hypothesis, let me know – I'll likely want you to present what you've discovered.

If you haven't done so yet, I encourage you to look up on Wikipedia/Internet: *Meno* (or read it – the Jowett translation is a classic and is free), Inquiry Based Learning, the Socratic Method. Non subject related things are worth looking up (sometimes.)

A final note: A standard calculation is to spend at least 2-3 hours per hour of class each week on homework (which in this class means 6-9 hours/week working on identifications and exercises) to maintain a reasonable average – A's and B's WILL require a bit more. One of the amazing things about the human mind, is that the subconscious can work on problems when the conscious mind is otherwise engaged (as in sleeping) – but in order for it to do that, the problems have to be firmly ensconced in the mind. Some of those hours where you feel that you getting "nowhere" still gives the subconscious information to work on; I hope you'll be delighted to have a eureka moment like Archimedes when a solution to a problem suddenly hits you when you're out jogging, enjoying your morning coffee or are otherwise preoccupied.