Prime Numbers and the Fundamental Theorem of Arithmetic.
Hints to selected theorems.

Theorem* 5.1. Suppose that each of a and b is a positive number. If $a|b$ and $b|a$ then $a = b$.

Hint: Use thm 3.9.

Theorem* 5.2. If $n > 1$ is a positive integer then there exists a prime number p so that $p|n$.

Hint. Use The 3.4 and consider two cases: (1) n is prime; (2) n is composite.

Theorem* 5.3. The set of prime numbers in infinite.

Hint. Argue that if p and$q$ are primes, then neither divides $pq + 1$.

Theorem* 5.8. Suppose that each of a and b is a positive integer and $d = gcd(a, b)$. Then there exists integers x and y so that:

$$d = ax + by.$$

Hint: Let $S = \{ax + by | x, y \in \mathbb{Z} \text{ and } (ax + by) > 0\}$ and use thm 3.4.

Theorem* 5.9. Suppose that each of a and b is an integer and at least one of them is not 0. Let $S = \{na + mb | n \in \mathbf{Z}, m \in \mathbf{Z}, 0 < na + mb\}$. Then $gcd(a, b)$ is the least element of the set S.

Hint: Let $S = \{ax + by | x, y \in \mathbb{Z}$ and $(ax + by) > 0\}$ and use thm 3.4.

Theorem* 5.10. Let a and b be integers at least one of which is not 0. Then a and b are relatively prime if and only if there exist integers x and y so that:

$$ax + by = 1.$$

Hint: Use Thm 5.8.

Theorem 5.18'. Suppose that p is a prime and n is a positive integer greater than one. Then there is a unique non-negative integer k so that $n = p^k q$ for some integer q and $p \nmid q$ (i.e. p does not divide q). [Note that, if not proven separately, this is also a corollary of the Fundamental Theorem of Arithmetic.]

Hint: Use Thm 5.2.