Square Symmetries Multiplication Table.

The set of "symmetries" of the square is listed below:

$$I : \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \qquad R : \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \to \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

$$R^2 : \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \to \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} \qquad R^3 : \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \to \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$V : \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \to \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \qquad H : \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \to \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$$

$$U : \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \to \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \qquad L : \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

To multiply two of them together you operate with one and then the other. You should convince yourself that this operation is not commutative; so it make a difference the order in which the operations are done. I'll base my notation on the way we are used to thinking about functions. So R times V means you operate first by V and then by R; in other words, as follows:

$$RV\left(\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\right) = R\Bigg(V\left(\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\right)\Bigg) = R\left(\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\right) = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}.$$

So $RV = U$.

Exercise 1. Use this information to construct the multiplication table for the symmetries of the square. [Hint: it's a group; so you can use the group properties to fill out the table more quickly (e.g it has an identity and inverses and it's a sudoku solution).]

Exercise 2. Show that the triangle symmetries is isomorphic to S_3.