

Logic Exercises

Notation: In the following, to simplify the notation, I sometimes use $P \vee \sim Q$ to mean $P \vee (\sim Q)$ and similarly with \wedge and \Rightarrow ; so $\sim P \Rightarrow Q$ means $(\sim P) \Rightarrow Q$.

Exercise 1. Determine if the following are theorems.

- a. $P \Rightarrow Q = (\sim Q \Rightarrow \sim P)$;
- b. $(\sim P) \Rightarrow Q = P \vee Q$;
- c. $\sim (P \vee \sim Q) = (\sim P) \vee (\sim Q)$;
- d. $\sim (P \vee \sim Q) = (\sim P) \wedge (\sim Q)$;
- e. $\sim (P \wedge \sim Q) = (\sim P) \vee (\sim Q)$;
- f. $\sim (P \wedge \sim Q) = (\sim P) \wedge (\sim Q)$;
- g. $(P \wedge Q) \vee R = P \wedge (Q \vee R)$;
- h. $(P \vee Q) \wedge R = P \vee (Q \wedge R)$.

Definition: A tautology is a statement that has truth value T no matter what are the truth value of the clauses making up the statement.

Exercise 2. Determine which of the following are tautologies.

- a. $P \vee (\sim P)$;
- b. $P \wedge (\sim P)$;
- c. $\sim (P \vee (\sim P))$;
- d. $P \Rightarrow P$;
- e. $\sim (P \wedge (\sim P))$;
- f. $(P \Rightarrow Q) \vee Q$;
- g. $(P \Rightarrow Q) \vee (\sim Q)$;
- h. $(P \Rightarrow Q) \vee P$;
- i. $(P \Rightarrow Q) \vee (\sim P)$;
- j. $(P \Rightarrow Q) \wedge Q$;
- k. $(P \Rightarrow Q) \wedge (\sim Q)$;
- l. $(Q \vee (P \vee R)) \vee (\sim R)$;
- m. $(Q \wedge (P \wedge R)) \wedge (\sim R)$.