

Induction Exercises.

Problem 1. Prove that for each positive integer n :

$$\sum_{i=1}^n (3i + 2) = \frac{n(3n + 7)}{2}.$$

Problem 2. Prove that for each positive integer n :

$$\sum_{i=1}^n i(i + 1) = \frac{n(n + 1)(n + 2)}{3}.$$

Problem 3. Prove that if $r > 0$ is a real number and $r \neq 1$ then the following holds for each positive integer n :

$$\sum_{i=0}^n r^i = \frac{1 - r^{n+1}}{1 - r}.$$

Problem 4. Prove that for each positive integer n :

$$\sum_{i=1}^n \frac{1}{(i + 2)(i + 3)} = \frac{n}{3n + 9}.$$

Problem 5. Prove that for each positive integer n :

$$\sum_{i=1}^n \frac{9}{10^i} = 1 - \frac{1}{10^n}.$$

Problem 6. Prove that for each positive integer n :

$$5 \mid (3^{4n} + 4).$$