Math 5000/6000 Project 01 Part 2.

Instructions: The project is due Wed. Sept. 14. You are allowed to work collaboratively with other members of the class, but with no other outside help. If you work in collaboration with anyone, you need to indicate who helped you and their main contribution toward your solutions; If you worked alone, please indicate this on your paper.

Show all your work; you may not receive full credit if the work is incomplete or incorrect. Indicate your reasoning, partial credit will be given if the reasoning is correct and only computational errors are made.

Problem 1. Two 1000 liter tanks connected in series are filled with pure water. At time $t = 0$ a 5 gm/liter solution of salt water starts flowing into the first tank at a rate of 25 liters/min. The salt water continues to flow into the tank at this rate and the well-mixed solution leaves the first tank and flows into the second tank at the same rate; then it (a well mixed solution) flows out of the second tank at the same rate.

a.) Find the concentration of salt in the first tank as a function of time.

b.) Find the concentration of salt in the second tank as a function of time.

c.) How long will it take for the concentration of salt in the first tank to reach 4 gm/liter.

d.) What is the limiting concentration of salt in the two tanks.

Problem 2. A mining operation that has been polluting a small lake was shut down by the EPA. When it was shut down, the lake had a pollutant at a concentration of 2.5 g/gal. An unpolluted stream enters the lake at a rate of 10,000 gal/day. Assume that the well-mixed solution flows out from the lake at an exit stream at the same rate and that there are no other streams affecting the amount of water in the lake. After a year the concentration of the lake is down to 1% of its original concentration. Calculate the volume of the lake.

Problem 3. A geologists is designing a seismograph. The instrument's design is based on the mechanical vibration of the stylus as a weight on a spring. The stylus moves when there is an earthquake and the stylus is given a sudden initial velocity. The mass of the stylus is 0.1 kg, the damping factor is 0.5

units and Hooke's constant for the spring engaging the stylus is 0.4. Before an earthquake happens the stylus is still. Suppose an earthquake occurs so that the stylus starts from an initial distance of 0 from it's equilibrium and it is given a sudden initial velocity of v m/sec.

i.) Set up the equation of motion for the stylus and solve the equation in terms of the initial velocity v.

ii.) Calculate when the stylus reaches its maximum displacement, in terms of v. Then calculate that displacement.

iii.) Is it possible to determine the initial velocity in terms of maximum displacement of the stylus? If no, say why; if yes calculate the initial velocity in terms of the distance (calculated in part ii) that the stylus traveled.

iv.) Suppose that there are two earthquakes and that the ratio of the initial velocities of the stylus of the first to the second is 3. What is the ratio of the peak distances of the stylus for the two earthquakes? Can you generalize for an arbitrary ratio r instead of 3?