Math 5000, Dr. Smith, Project 02.

We will have a test next Friday Oct. 28, the project is due before the test.

Make sure to show all your work. You may not receive full credit if the accompanying work is incomplete or incorrect. If you do scratch work make sure to indicate scratch work - I will not take off points for errors in the scratch work if it is so labeled. Make sure to distinguish between scratch work and your solution.

You may collaborate with other students.

Situation.

An ecologist wishes to examine in detail the modeling of the interaction of two species populations based on a system of differential equations. [We will base the modeling on one of our textbook examples.]

Questions.

Population Model # 1. Consider the predator-prey model given below, where x denotes the prey and y the predator populations [see example 1 pg. 429.]

$$\begin{aligned} x' &= x(1 - 0.5y) \\ y' &= y(-0.75 + 0.25x). \end{aligned}$$

Our ecologists want to determine how alternating the various constants affects the population. For each of the following constants, $\alpha, \beta, \gamma, \delta$, in the systems described below, determine the range of the constants that allows both species to limit to non-zero populations:

Problem a.

$$\begin{aligned} x' &= x(\alpha - 0.5y) \\ y' &= y(-0.75 + 0.25x). \end{aligned}$$

Problem b.

$$\begin{aligned} x' &= x(1 - \beta y) \\ y' &= y(-0.75 + 0.25x). \end{aligned}$$

Problem c.

$$\begin{aligned} x' &= x(1 - 0.5y) \\ y' &= y(\gamma + 0.25x). \end{aligned}$$

Problem d.

$$\begin{aligned} x' &= x(1 - 0.5y) \\ y' &= y(-0.75 + \delta x). \end{aligned}$$

In each case determine if there is an optimal value for the parameter. The term "optimal" can be interpreted in different ways. Let's consider three types of optimal, where we consider the highest populations of each of the two and then some reasonable combination of the two.