Anotes05
Compact Sets.

Definition. If M is a set and G is a collection of sets, then G is said to cover M if and only if each point of M lies in some element of G.

Definition. If M is a set, then M is said to be compact if and only if for each collection G of segments covering M there is a finite subcollection G' of G that covers M.

Exercise 5.1. Show that a segment is not compact; show that the set of real numbers is not compact.

Theorem 5.1. If M is a finite set then M is compact.

Exercise 5.2. Show that:

a. An unbounded set is not compact.

b. A closed and bounded set with one limit point is compact.

c. A closed and bounded set with a finite collection of limit points is compact.

Theorem 5.2. The set M is compact if and only if for each collection G of open sets covering M there is a finite subcollection G' of G that covers M.

Theorem 5.3. If M is compact then M is bounded.

Theorem 5.4. If M is compact then M is closed.

Theorem 5.5. If M is the interval $[a, b]$ then M is compact.

Theorem 5.6. If M is compact and G is a monotonic collection of closed subsets of M then there is a point P that is common to all the elements of G. (Equivalently: $\cap G \neq \emptyset$.)

Definition. The set M is said to be bounded if there exists a number $B > 0$ so that $M \subset [-B, B]$.

Theorem 5.7 If M is a closed and bounded set then M is compact.