Math 5200/6200, Dr. Smith
Test Prep Exercises

Problem 1. Prove, from the definition of continuity, that the function $f(x) = x^3$ is continuous at the point $(p, f(p))$.

Problem 2. Let $p > 0$. First let p be a fixed number and calculate the following limit:

$$\lim_{h \to 0} \frac{(p+h)^2 - p^2}{h}.$$

[An "educated" guess from your calculus class is fine.] Then let L denote that limit and let $g(x)$ be defined as follows:

$$g(x) = \begin{cases} \frac{(p+x)^2 - p^2}{x} & \text{if } x \neq 0 \\ L & \text{if } x = 0. \end{cases}$$

Prove that the function g is continuous at $x = 0$.

Problem 3. Prove, from the definition, that the function $f(x) = x^3$ is uniformly continuous over the interval $[0, 2]$.

Problem 4.Prove, from the definition, that the function $f(x) = \frac{1}{3x^2+4}$ is continuous.

Problem 4. Prove, from the definition, that the function $f(x) = \sqrt{x}$ is continuous at $(4, 2)$.

Problem 5. For each integer n let

$$f_n = 2x + \left(\frac{x}{3}\right)^n.$$

Show:

a. That the sequence $\{f_n\}_{n=1}^{\infty}$ of functions approach a function g pointwise over $[0, 3]$.

b. That g is not continuous at $x = 3$.

c. That $\{f_n\}_{n=1}^{\infty}$ approaches g uniformly over the interval $[0, 2]$.

General Problems: Proofs of some of the theorems or lemmas proven in class.