MATH 5200 Take Home Project 02.

The project is due Monday Feb. 21 by the beginning of class. You are allowed to work together, but if you do you must indicate which person (or persons) you worked with and the critical contributions of that person (or those persons.)

Provide solutions to all the problems, I will base the grade on the best 5 out of 6.

As usual, email to me a pdf copy of your work with your last name as the first part of the file name.

1. Prove that the compliment of an open set is a closed set.

2. Consider the sequence

$$S = \left\{ \frac{2\sqrt{n} - 5}{\sqrt{n} + 2} \,\middle|\, n \in \mathbb{N} \right\}.$$

Find the sequential limit point of S and prove that the point you found is the sequential limit point of S.

3. Consider the following sequence

$$S = (-1)^n \left\{ \frac{1}{5} + \frac{7}{n} \right\}_{n=1}^{\infty}.$$

a.) Prove that $\frac{1}{5}$ is a limit point of the set of terms of the sequence.
b.) Prove that the number $\frac{1}{5}$ is not the sequential limit of this sequence.
c.) Show that the sequence does not have a sequential limit.

[Note that doing (c) implies the solution to (b) - I set it up this way because working through (b) should help you do (c).]

4.) Prove, from the definition of sequential limit point, that if a is the sequential limit point of the sequence $\{a_n\}_{n=1}^{\infty}$ then a^2 is the sequential limit point of the sequence $\{a_n^2\}_{n=1}^{\infty}$.

[Hint 1: $a^2 - a_n^2 = (a - a^n)(a + a_n)$.]

[Hint 2: Prove that the set $M = \{a + a_n | n \in \mathbb{Z}^+\}$ is bounded and look at the proof of theorem 3.4.]

5.) Suppose that $\{M_i\}_{i \in I}$ and that for each $i \in I$, L_i is the least upper bound of M_i. Prove that if L is the least upper bound of $\cup_{I \in I} M_i$, then L is the least upper bound of the set $\{L_i | i \in I\}$.

6.) Prove theorem 4.2.
[Hint: the proof requires the least upper bound axiom.]