

Math 5200/6200
Quiz 01

Scan (or photograph) your work; email it to me before class on Wednesday March 23; send your work to me as a pdf file whose name begins with your last name (e.g. smith_quiz_1). I will use problems 1-3 to help increase your project grades.

Since this is a quiz, you may not give or receive outside help on the problems.

For these problems, you must do them from the definition of continuity. You may also use the following lemma about continuous functions and boundedness:

Lemma: If the function f is continuous at the point $(p, f(p))$ then there exists a number B and a number δ so that $|f(x)| < B$ for all $x \in (p - \delta, p + \delta)$.

Problem 1. Prove, from the definition of continuity, that the function $f(x) = 5x^2 - 3x^3$ is continuous at the point $(p, f(p))$ for $p > 0$.

Problem 2. Prove, from the definition of continuity, that the following function is continuous at the point $(2, 5)$:

$$f(x) = \begin{cases} 2x + 1 & \text{if } x \leq 2 \\ 9 - x^2 & \text{if } 2 < x \end{cases}$$

Problem 3. Consider the following function:

$$f(x) = \begin{cases} 4x & \text{if } x \leq 1 \\ \frac{10-x}{2} & \text{if } 1 < x \end{cases}$$

Determine whether or not f is continuous and prove your conclusion.

Problem 4. Prove theorem 6.2 or 6.3 from the notes.