MATH 5200 Take Home Project 03.
Part A

Part A of Project03 is due Friday April 22 by the beginning of class. You are allowed to work together, but if you do you must indicate which person (or persons) you worked with and the critical contributions of that person (or those persons.)

As usual, email to me a pdf copy of your work with your last name as the first part of the file name.

The Definition of the Limit.

Let f be a function $f : \mathbb{R} \to \mathbb{R}$.

Definition 1. *The limit of a function f.*

$$L = \lim_{x \to p} f(x)$$

means that for each positive number ϵ there is a number δ_ϵ so that:

$$\text{If } 0 < |x - p| < \delta_\epsilon \quad \text{then} \quad |f(x) - L| < \epsilon.$$

Problem 1 For the following functions, determine the indicated limit if it exists. If the limit exists, prove that your choice of the limit is correct from the definition; if the limit does not exist, prove this.

a.) $\lim_{x \to 2} x^2$

b.) $\lim_{x \to 4} \sqrt{x}$

c.) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

d.) $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

e.) $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$

f.) $\lim_{h \to 0} \frac{|h|}{h}$.

Prove the following theorems.

Theorem A: Let $p \in (a,b)$. Then the function $f : (a,b) \to \mathbb{R}$ is continuous at $(p, f(p))$ if and only if $\lim_{x \to p} f(x) = f(p)$.

Theorem B: If a and b are numbers and f and g are functions $f, g : \mathbb{R} \to \mathbb{R}$ so that

$$\begin{aligned} \lim_{x \to p} f(x) &= K \\ \lim_{x \to p} g(x) &= L. \end{aligned}$$

Then

$$\lim_{x \to p} af(x) + bg(x) = aK + bL.$$