Useful Limit Theorems.

Reminder: In the following assume all functions have domain some open set and range a subset of the reals. Assume also that each formula is well-defined (e.g. p is in the common domain where required.) [There are similar theorems for sequential limits.]

Definition. Suppose $f : U \to \mathbb{R}$ is a function and p is a number in the domain of f. Then

$$\lim_{x \to p} f(x) = L$$

means that if $\epsilon > 0$ then there exists a number δ so that

$$|f(x) - L| \; < \; \epsilon$$

for all x so that

$$0 \; < |x - p| < \; \delta.$$

In the following theorems assume all functions have the number p in their domains.

Theorem L1.

$$\lim_{x \to p}(f(x) + g(x)) = \lim_{x \to p} f(x) + \lim_{x \to p} g(x).$$

Theorem L2. If c is a constant then

$$\lim_{x \to p}(cf(x)) = c \lim_{x \to p} f(x).$$

Theorem L3.

$$\lim_{x \to p}(f(x) \cdot g(x)) = \lim_{x \to p} f(x) \cdot \lim_{x \to p} g(x).$$

Theorem L4. If

$$\lim_{x \to p} f(x) \neq 0$$

then

$$\lim_{x \to p} \frac{1}{f(x)} = \frac{1}{\lim_{x \to p} f(x)}.$$