Exercises on Bounded Variation

Exercise 1. Calculate $V_0^x f$ for each of the following functions f:

(a) $f(x) = (x-1)^2$;

(b) $f(x) = x(x-1)(x-2)$;

(c) (d) $g(x) = \begin{cases} 2x & \text{if } x \leq 1 \\ 3x & \text{if } 1 < x. \end{cases}$

Exercise 2. For each of exercises a - d below, calculate $\int_0^2 f \, dg$ for (i) $f = 5x$ and for (ii) $f = x^2$ where g is given below; then verify that your calculation is correct from the definitions:

(a) $g = x^2$;

(b) $g(x) = \begin{cases} 0 & \text{if } x \leq 1 \\ 1 & \text{if } 1 < x; \end{cases}$

(c) $g(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1 & \text{if } 1 \leq x; \end{cases}$

(d) $g(x) = \begin{cases} 2x & \text{if } x \leq 1 \\ 3x & \text{if } 1 < x. \end{cases}$

Exercise 3. Determine if the following function is (a) R-integrable over $[0, 1]$, (b) of bounded variation over $[0, 1]$:

$$f(x) = \begin{cases} 1 & \text{if } x = \frac{1}{n} \text{ for } n \in \mathbb{Z}^+ \\ 0 & \text{elsewhere.} \end{cases}$$