Hints on Integration Exercises and Theorems

Exercise 4.3c. Observe that

$$\begin{aligned} x_i^2 - x_{i-1}^2 &= (x_i + x_{i-1})(x_i - x_{i-1}) \\ &= 2\frac{x_i + x_{i-1}}{2}(x_i - x_{i-1}) \\ &= 2c_i^*(x_i - x_{i-1}). \end{aligned}$$

Now you've got an x^* and a c^*; figure out how to address that. Recall my short lecture today (Mar. 11) about this.

Mean Value Theorem Review Exercise.

If $f = rx^2$ for some constant r, then find the number $c \in [a, b]$ that satisfies the Mean Value Theorem (Thm. 1.5.) In other words the number c with $a < c < b$ so that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Theorem 4.2.

First show that the integral exists. To do this, consider stating (and proving) a lemma similar to Theorem 3.3'. Do the special case of g increasing and continuous first.

Then to get the $\int fg'$ formula use the mean value theorem.

Theorem 2.5.

This theorem now follows from some theorems we've already done: We know that F is continuous; that f attains it max and min values in an interval; and recall the intermediate value theorem.

Theorem 2.4c.

Observe that we didn't use theorem 2.4c to prove 2.5, but (possibly) only 2.4a. So try assuming 2.5.

Bounded Variation Exercise.

Let $f(x) = x(1-x)$. Conjecture: The function $f(x) + \mathcal{V}_0^x f$ is non-decreasing.

Lemma on Bounded Variation:

For a function $f : [a,b] \to \mathbb{R}$, if $x < y$ we have,

$$\mathcal{V}_a^y f = \mathcal{V}_a^x f + \mathcal{V}_x^y f.$$

Theorem 4.5.

Examine the function:

$$\mathcal{V}_a^x f + f(x).$$

Theorem 4.7.

Argue that $\int f d(g_1 + g_2) = \int f dg_1 + \int f dg_2$ and consider previous theorems and exercises.