Quiz02.

The quiz is due before by class time, 12:30 pm February 02. Quiz may be handwritten and should be converted to a pdf file which is to be emailed to me with the file name beginning with your last name.

Make sure to show all your work. You may not receive full credit if the accompanying work is incomplete or incorrect. If you do scratch work make sure to indicate scratch work - I will not take off points for errors in the scratch work if it is so labeled. Make sure to distinguish between scratch work and proof.

Note that all the proofs must follow logically from the theorems and definitions stated in the class notes; if you wish to use some lemma that has not be proven in class, you must prove it first using the theorems and definitions stated from the class notes.

In all these problems assume that any require quantities exist. (E.g. if I have $f'(p)$ as part of a formula, then assume that f is a function differentiable at p.)

Problem 1. Suppose:

$$\lim_{x \to p} f(x) = q \quad \text{and} \quad \lim_{x \to q} g(x) = r.$$

Then use the $\epsilon - \delta$ definition of limit to show that

$$\lim_{x \to p} g(f(x)) \quad = \quad r.$$

Problem 2. Suppose $f : \mathbb{R} \to \mathbb{R}$ and that for all $x \in \mathbb{R}$ we have:

$$-x^2 \le f(x) \le x^2.$$

Use the $\epsilon - \delta$ definition of limit to prove that $f'(0) = 0$.

Problem 3. Prove the Lemma to Theorem 1.5. (I just posted it.)