MATH5310 Dr. Smith Test 1 prep.

1. Suppose that for $x, y \in \mathbb{Z}$ the relation $\sim$ is defined by $x \sim y$ if and only if $5|4y + x$.

   a.) Show that $\sim$ is an equivalence relation.

   b.) List the elements of [7], the equivalence class of 7.

2. Define $f : (\mathbb{Z}_{20}, +_{20}) \to (Z_{10}, +_{10})$ by $f([x]_{20}) = [12x]_{10}$.

   a.) Show that f is well defined.

   b.) Is f one-ton-one?

   c.) Is f onto?

   e.) Show that f is a homomorphism.

   f.) Find the kernel of f.

3. Let $G = (Z_{18}, +_{18})$ and $H = \{0, 6, 12\}$ (where I'm using the shorthand 6 for $[6]_{18}$, etc.) Show that G/H is isomorphic to $(Z_6, +_6)$.

4. Exercises 1 and 2 of AlgebraNotes01.

5. Make sure you can do Theorems 03, 04 and 05 also Lagrange's theorem (Theorem 06); also theorems 3.3 -3.6.

6. Consider $G = (\mathbb{Z}_{100}, \cdot_{100})$.

   a.) Argue that G is not a group.

   b.) Show that although G is not a group, G has an identity and that $[59]_{100}$ and $[39]_{100}$ are inverses of one another.

   c.) Let $f : \mathbb{Z}_{100} \to \mathbb{Z}_{100}$ be defined by $f([x]_{100} = [59x + 17]_{100}$. Show that f is well-defined.

   d.) Use (b) to prove that the function f is one-to-one and onto.

7. Prove for $(\mathbb{Z}_n, \cdot_n)$ that $[n - 1]_n$ is always its own inverse.

8a. Prove that if G is a group, $g \in G$ and $H = \{g^n |\ n \in \mathbb{Z}\}$. Then H is a cyclic subgroup of G.

8b. Prove that if G is a finite group, $|G| = p$ and p is a prime number, then G is a cyclic group and therefore abelian.