Math 5500/6500, Fall 2021, Dr. Smith
Test 01; Problem 4 Solution

Problem 4. Consider the following subsets M_i of $\mathbb{R}$:

$$\begin{aligned} M_1 &= \left\{1-\frac{1}{n}\right\}_{n=1}^{\infty} \bigcup \left\{2+\frac{2}{n}\right\}_{n=1}^{\infty} \bigcup \left\{3-\frac{4}{n}\right\}_{n=1}^{\infty} \\ M_2 &= \{x \mid x^2 < 5\} \cup [5,7] \\ M_3 &= \{x \mid x \leq -7\} \cup ((2,3) \cap \mathbb{Q}) \cup (5,6). \end{aligned}$$

Part A. Using the standard topology of $\mathbb{R}$, determine
(i) the set of limit points of M_i;
(ii) the interior of M_i;
(iii) the boundary of M_i.
Part B. Repeat part A but with the topology $\widehat{\mathcal{T}}$ on $\mathbb{R}$ generated by the basis $\widehat{\mathcal{B}} = \{[a,b) \mid a < b\}$.

Solution. Let's use the following notation:
(i) M_i' denotes the set of limit points of M_i;
(ii) $\text{int}(M_i)$ denotes the interior of M_i;
(iii) $\text{Bd}(M_i)$ denotes the boundary of M_i.

Part A:

$$\begin{aligned} M_1' &= \{1,2,3\} \\ M_2' &= [-\sqrt{5},\sqrt{5}] \cup [5,7] \\ M_3' &= (-\infty,-7] \cup [2,3] \cup [5,6] \end{aligned}$$

$$\begin{aligned} \text{int}(M_1) &= \emptyset \\ \text{int}(M_2) &= [-\sqrt{5},\sqrt{5}) \cup (5,7) \\ \text{int}(M_3) &= (-\infty,-7) \cup (5,6) \end{aligned}$$

$$\begin{aligned} \text{Bd}(M_1) &= M_1 \cup \{1,2,3\} \\ \text{Bd}(M_2) &= \{-\sqrt{5},\sqrt{5},5,7\} \\ \text{Bd}(M_3) &= \{-7,5,6\} \cup [2,3] \end{aligned}$$

Part B:

$$
\begin{aligned}
M_1' &= \{2\} \\
M_2' &= [-\sqrt{5}, \sqrt{5}) \cup [5,7) \\
M_3' &= (-\infty, -7) \cup [2,3) \cup [5,6)
\end{aligned}
$$

$$
\begin{aligned}
\text{int}(M_1) &= \emptyset \\
\text{int}(M_2) &= [-\sqrt{5}, \sqrt{5}) \cup [5,7) \\
\text{int}(M_3) &= (-\infty, -7) \cup (5,6)
\end{aligned}
$$

$$
\begin{aligned}
\text{Bd}(M_1) &= M_1 \cup \{2\} \\
\text{Bd}(M_2) &= \{-\sqrt{5}, 7\} \\
\text{Bd}(M_3) &= \{5, \} \cup [2,3)
\end{aligned}
$$

$\square$