Math 5630/6630 Project01.

Instructions: The projet is due before class Wed. June 21. You are allowed to work collaboratively with other members of the class, but with no other outside help. If you work in collaboration with anyone, you need to indicate who helped you and their main contribution toward your solutions. Turn in your code as well as the output of your programs. You may submit the project electronically (I like pdf files); if you submit your work electronically please begin the file name with your last name (e.g. smith-project01.pdf).

For exercises 1 and 2 use each of the following methods to find and approximate all the roots of the given function:

(a.) bisection method,
(b.) Newton's method,
(c.) secant method (start with two iterates that are 0.1 apart).

In each case stop the program when the approximation of the root x_n is such that $|f(x_n)| < 10^{-10}$; print out the approximation of the root and the number of iterates n that were needed. Stop the program in case of failure after 100 iterates.

Exercise 1. $f(x) = x^4 + 3x - 7$.

Exercise 2. $f(x) = x^3 - 3x^2 + 3.9$.

Exercise 3. Consider the set of data points: $(1, 1.5), (2, 2), (2.5, 2.1), (3.5, 1.9)$ we wish to find the lowest degree polynomial $p(x)$ that goes through these points.

a.) Find the coefficients for the Lagrange form of the polynomial.

b.) Find the coefficients for the Newton form of the polynomial.

c.) Evaluate $p(3.0)$. [Note it might not be a bad idea to check to see that the forms from parts a and b give the same value.]

d.) Add the data point $(4, 1.7)$ and evaluate $p(3.0)$. (You may use what ever form of the polynomial you wish.)

Exercise 4. Find the cubic spline $S(x)$ that goes through the following points $(1, 1.5), (2, 2), (2.5, 2.1), (3.5, 1.9), (5, 1)$.

a) Use the “natural” method.

b) Use the “not-a-knot” method.

c) Calculate $S(3)$ using each of the spines obtained in (a) and (b).

d) Have the computer plot the splines from both methods on the same page.

Exercise 5. Consider the following function:

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq \pi \\ 0 & \text{if } \pi < x \leq 2\pi \end{cases}$$

a) Find the Fourier series for f. [See formulas 13.1 a, b & c except integrate from 0 to 2π.] There is a pattern to the coefficients so you should be able to express it as an infinite sum.

b) Plot the series up to the $n = 10$ term.