MATH5630/6630 Dr. Smith, Formula Sheet.

1.) Divided Difference Table: The rows are number $k = 0, 1, 2, 3, \ldots$ and there is a column of the x values (column # -1 ?) after which the columns are numbered $\ell = 0, 1, 2, 3, \ldots$.

$$\begin{array}{llllllll}
x_0 & y_0 & & & & & \\
x_1 & y_1 & a_1 & & & & \\
x_2 & y_2 & a_2 & b_2 & & & \\
x_3 & y_3 & a_3 & b_3 & c_3 & & \\
x_4 & y_4 & a_4 & b_4 & c_4 & d_4 & \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ldots
\end{array}$$

where the entries are calculated as follows:

$$\begin{array}{llllllll}
x_0 & y_0 & & & & & \\
x_1 & y_1 & \frac{y_1-y_0}{x_1-x_0} & & & & \\
x_2 & y_2 & \frac{y_2-y_1}{x_2-x_1} & \frac{a_2-a_1}{x_2-x_0} & & & \\
x_3 & y_3 & \frac{y_3-y_2}{x_3-x_2} & \frac{a_3-a_2}{x_3-x_1} & \frac{b_3-b_2}{x_3-x_0} & & \\
x_4 & y_4 & \frac{y_4-y_3}{x_4-x_3} & \frac{a_4-a_3}{x_4-x_2} & \frac{b_4-b_3}{x_4-x_1} & \frac{c_4-c_3}{x_4-x_0} & \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ldots
\end{array}$$

2.) Polynomial Approximation Error:

Suppose $f \in C^{[n+1]}[a, b]$ and $P(x)$ is a polynomial approximation for $f(x)$ that contains the points $\{(x_i, y_i)\}_{i=0}^{n}$. Then for any x between $\min\{x_0, x_1, \ldots, x_n\}$ and $\max\{x_0, x_1, \ldots, x_n\}$ there exists a number ξ_x also between $\min\{x_0, x_1, \ldots, x_n\}$ and $\max\{x_0, x_1, \ldots, x_n\}$ so that

$$f(x) = P(x) + \frac{f^{[n+1]}(\xi_x)}{(n+1)!}(x - x_0)(x - x_1) \ldots (x - x_n).$$

3.) Taylor's Theorem. Suppose that $f \in C^{[n+1]}[a,b]$ then for each $x \in (a,b)$ there exists number $\xi_x \in (a,b)$ so that:

$$f(x) = f(a)+f'(a)(x-a)+f''(a)\frac{(x-a)^2}{2!}+\ldots+f^{[n]}(a)\frac{(x-a)^n}{n!}+f^{[n+1]}(\xi_x)\frac{(x-a)^n}{n!}.$$