Topology 7500, Fall 2023
Dr. Michel Smith
Hand In Project 2, Part 1

The final due date is TBA. For Wednesday I'd like everyone to just send me a list of which properties from problem 2 part I and part II that they think are preserved under the given conditions. No proofs are necessary for Wednesday. I'd also like someone to present as much of problem 1 that they have figured out (e.g. give an example of what can happen).

You are permitted to collaborate with other students from the class, but you must indicate for which problems you did not collaborate with anyone and you must indicate with whom you collaborated when you did so (and for which problems).

Problem 1. Consider the upper half plane $\{(x, y)|x, y \in \mathbb{R}, y \geq 0\}$ with the tangent disc topology. Recall that the x-axis, with the subspace topology (of the tangent disc topology), has the discrete topology. Let $Q = \{(x, 0)|x \text{ is rational }\}$ and $P = \{(x, 0)|x \text{ is irrational }\}$. Suppose that U and V are disjoint open sets containing the Q and P respectively.

a.) Argue that for each $(x, 0)$ there is a δ_x and a tangent disc $R_x = \{(x, y)|x^2 + (y - \delta_x)^2 < \delta_x^2\}$ with radius δ_x that lies in U or V (depending on whether or not x is rational).

b.) Let $\epsilon > 0$. Show that, with respect to the usual topology on the x-axis that the set $\{(x, 0) \in P|\delta_x > \epsilon\}$ is a no-where dense subset of the x-axis.

c.) Argue that condition (b.) leads to a contradiction.

Problem 2. (See exercise 3.6.) Suppose that each of X and Y is a topological space and $f : X \to Y$ is a continuous onto function.

I. Determine if it is true that if X is has property $\mathcal{P}$ then Y must have property $\mathcal{P}$ where property $\mathcal{P}$ is:

a. Hausdorff.
b. Regular.
c. Normal.
d. A Moore space.
e. First countable.
f. Compact.
g. Separable.

h. Completely separable.

II. Suppose in addition that f is one-to-one and that f^{-1} is also continuous. Again, determine if it is true that if X is has property $\mathcal{P}$ then Y must have property $\mathcal{P}$

[Hint. Prove the following lemma: If X is discrete and $f : X \to Y$, then f is continuous.]