Localized viscous heating observed in a two-dimensional strongly coupled dusty plasma

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Shear flow in 2D dusty plasma
Shear flow in dusty plasma
Fluids, in general

Transport coefficients

\[ \mu = \text{viscosity} \]

\[ \kappa = \text{thermal conductivity} \]

These coefficients are material properties.
Viscous heating: it’s where a lot of energy is lost
Viscous heating happens in a sheared flow

Couette flow between two flat plates
Viscous heating happens in a sheared flow

“shear”
  = transverse gradient of a flow velocity
  = $\partial v_x / \partial y$

Heat generated $\propto (\partial v_x / \partial y)^2$
Hydrodynamic equations for Couette flow

**Momentum**
\[ \frac{\eta}{\rho} \nabla^2 \mathbf{v} = 0 \]

**Energy**
\[ \frac{\kappa}{\rho} \nabla^2 T + \frac{\eta}{\rho} \left( \frac{\partial v_x}{\partial y} \right)^2 = 0 \]

\( \mu = \) viscosity
\( \kappa = \) thermal conductivity

thermal conduction

viscous heating
Solution of the hydrodynamic equations for Couette Flow

Solutions:
\[ \nu_x \propto y \]
\[ T \propto y^2 \text{ due to viscous heating} \]

viscous heating:
- should cause **hot-spots** in a flow
- but they aren’t observed
Why aren’t the hot spots observed?

For most substances:

thermal conductivity $\kappa$ is so large that $\nabla^2 T$ is tiny.

To observe a hot spot requires a combination of:

- Small $\kappa$ thermal conductivity
- Large $\eta$ viscosity
- Large $\partial v_x/\partial y$ shear
- Instrumentation for in-situ temp measurement
Experimental setup

**Argon RF glow discharge:**
- gas: 15.5 mTorr Argon
- RF: low power, 13.6 MHz
- $\lambda_D$: 0.53 mm

**MF Polymer microspheres:**
- diameter: 8.1 $\mu$m
- number: >11000 particles
- interparticle dist.: 0.50 mm
- charge: -9700 e
Narrow field of view

Random motion in a crystalline phase
Measurements of particles

- Measure particle \(x,y\) positions

\[
X = \frac{\sum (X_i I_i)}{\sum I_i}
\]

- Track particles from frame to frame

Intensity in pixel \(i\) is \(I_i\)

Yields time series for positions & velocities of particles
Manipulating particles using Radiation Pressure Force

\[ F \propto I_{laser} \]
Movie of the laser-driven shear flow

laser 1

laser 2

y

x
Results: profiles of flow velocity & temperature

flow velocity profile $v_x$

regions of high shear

temperature profile $T$

Temperature peaks due to viscous heating
Confirm the conclusion

To confirm that the peaks are due to viscous heating:

• Calculate the Brinkmann number

\[
Br \equiv \frac{\eta \Delta v_x^2}{\kappa \Delta T}
\]

• Confirm that it is of order unity

We require:

\[
\begin{align*}
\Delta v_x & \quad \text{Flow parameters} \\
\Delta T & \\
\eta & \quad \text{Material parameters} \\
\kappa &
\end{align*}
\]
Calculate the Brinkmann number

Measure the flow parameters:

\[ \Delta v_x \]
\[ \Delta T \]
Calculate the Brinkmann number

Measure the material properties:

\( \eta \)
\( \kappa \)

We do this by fitting the profiles to the hydrodynamic equations.
Calculate the Brinkmann number

We find the Brinkmann number

\[ \text{Br} \equiv \frac{\eta}{\kappa} \frac{\Delta x}{\Delta T} \approx 0.5 \]

Since \( \text{Br} \approx \text{unity} \), this result confirms the conclusion:

We have observed spatially-localized viscous heating.

Our temperature peaks are the elusive “hot spots” of viscous heating.
Observation of Temperature Peaks due to Strong Viscous Heating in a Dusty Plasma Flow

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Profound temperature peaks are observed in regions of high velocity shear in a 2D dusty plasma experiment with laser-driven flow. These are attributed to viscous heating, which occurs due to collisional scattering in a shear flow. Using measurements of viscosity, thermal conductivity, and spatial profiles of flow velocity and temperature, we determine three dimensionless numbers: Brinkman, Br = 0.5; Prandtl, Pr = 0.09; and Eckert, Ec = 5.7. The large value of Br indicates significant viscous heating that is consistent with the observed temperature peaks.
Second derivative of the flow velocity

\[ \frac{d^2 v_x(y)}{dy^2} \]
Second derivative of the kinetic temperature
Temperature dependent thermal conductivity $\kappa(T)$

All transport coefficients are temperature dependent, for example $\kappa$

$$\frac{\eta}{2\rho} \left( \frac{\partial v_x}{\partial y} \right)^2 + \left( \frac{1}{\rho} \right) \frac{\partial}{\partial y} \left( \kappa \frac{\partial T}{\partial y} \right) - \nu \bar{v}^2 = 0$$

$$\left( \frac{\eta}{2\rho} \right) \left( \frac{\partial v_x}{\partial y} \right)^2 + \left( \frac{\kappa}{\rho} \right) \frac{\partial^2 T}{\partial^2 y} + \left( \frac{1}{\rho} \right) \frac{\partial T}{\partial y} \frac{\partial \kappa}{\partial y} - \nu \bar{v}^2 = 0$$

Temperature variation is not extreme, so we can assume $\kappa = \kappa_0 + \alpha T$

$$\left( \frac{\eta}{2\rho} \right) \left( \frac{\partial v_x}{\partial y} \right)^2 + \left( \frac{(\kappa_0 + \alpha T) - \rho}{\rho} \right) \frac{\partial^2 T}{\partial^2 y} + \alpha \left( \frac{1}{\rho} \right) \left( \frac{\partial T}{\partial y} \right)^2 - \nu \bar{v}^2 = 0$$
**Temperature dependent thermal conductivity $\kappa(T)$**

Minimizing the residual of the energy equation in no laser region

$$\text{Residual} \left( \kappa \right) = \left( \eta / 2 \rho \right) \left( \partial v_x / \partial y \right)^2 + \left[ \left( \kappa_0 + \alpha T \right) / \rho \right] \partial^2 T / \partial^2 y + \alpha \left( 1 / \rho \right) \left( \partial T / \partial y \right)^2 - \nu v^2$$

For the region between flows:

$$\frac{\kappa(T)}{\rho} = (7.17 + 1.12 \nu_{\text{thermal}}^2) \text{mm}^2 / \text{s}$$

$$\nu_{\text{thermal}}^2 \in [0.42, 0.84]$$

$$\frac{\kappa(T)}{\rho} \in [7.64, 8.11]$$

$$\frac{\kappa}{\rho} = 8.3 \text{ mm}^2 / \text{s}$$
Transport coefficients using previous methods

<table>
<thead>
<tr>
<th>Our method of minimizing residuals</th>
<th>Previous methods</th>
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<tbody>
<tr>
<td></td>
<td>Nosenko et al., PRL (2008)</td>
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<tr>
<td>$\eta/\rho$</td>
<td></td>
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<tr>
<td>$\kappa/\rho$</td>
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<tr>
<td>$\kappa(T)/\rho$</td>
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The transport coefficients using our method are consistent with those using different methods.
Validity of transport coefficients in 2D systems

Some theorists argue that transport coefficients in 2D system does not exist, due to non-converging integrals of the Green-Kubo relation for uniform system.

For non-uniform systems with a gradient, the analogous test would be to determine whether transport coefficients is independent of the scale length.

But, our experiment cannot give a clear answer to this question.
Movie of the laser-driven flow in dusty plasma

laser 1

analyzed region

laser 2
Parameters for laser manipulation

**Manipulation laser:**

- Power: 1.14 W for each
- Beam width: ≈ 0.2 mm
- Incident angle: 6°
- Repetition rate in y: 200 Hz
- Repetition rate in x: 123.6067977 Hz
Data analysis: characterize nonuniformity in $y$

1. Divide the field of view into 89 bins (of width $\Delta d = r_{ws}$)

2. For each frame, **bin** the information (e.g. velocity) of all particles in a stripe
   - Each particle contributes to two stripes, according to a “weight”
   - weighting factor: Cloud-in-Cell algorithm
Scheme for producing counterpropagating flows

F_laser is zero everywhere except within two stripes
What is viscosity?

Viscosity (\(\eta\))

A measure of how a fluid flows under the shear stress.

\[
\eta = \text{shear stress} \frac{\partial u}{\partial y}
\]

kinematic viscosity = \(\frac{\eta}{\rho}\)
**Measuring \( \eta \) with shear: Setup**

Experiment setup


Two laser beams:
apply shear by two counter propagating flows

shear-induced melting
Measuring $\eta$ with shear: Movie

Movie of suspension with high shear
Measuring $\eta$ with shear

Particle trajectories

256 frames averaged in steady-state regime
Measuring \( \eta \) with shear: Result

Model the velocity profile to continuum, using Navier-Stokes equation

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \frac{\nabla p}{\rho} + \frac{\eta}{\rho} \nabla^2 \mathbf{v} + \left[ \frac{\xi}{\rho} + \frac{\eta}{3\rho} \right] \nabla (\nabla \cdot \mathbf{v}) - \nu_{gas} \mathbf{v}
\]

\[
\eta \frac{\partial^2 v_x(y)}{\partial y^2} - \nu_{gas} v_x(y) = 0
\]

\[
v_x(y) = \sqrt{\frac{\eta}{\rho}} \frac{\sinh(\alpha y)}{\sinh(\alpha h)}
\]

\[
\alpha = \sqrt{\nu_{gas} \rho / \eta}
\]

\( h \) is the width of between flows
Shear viscosity determination in our experiment

Flow velocity profile $\langle v_x \rangle$

$\nu_x(y) = V \frac{\sinh(\alpha y)}{\sinh(\alpha h)}$

$\alpha = \sqrt{\nu_{gas} \rho \eta}$

$h$ is the width of between flows
Shear viscosity determination in our experiment

\[ v_x(y) = \sqrt{\sinh(\alpha y) / \sinh(\alpha h)} \]

The fitting result of \( \alpha = \sqrt{\nu_{\text{gas}} \rho / \eta} \)

is not affected by the input value of \( h \)

The determined viscosity is

\[ \eta / \rho = 0.8 \ mm^2 / s \]
What is thermal conductivity?

Thermal conductivity \((\kappa)\)

The property of a material’s ability to conduct heat.

Heat flux \(\dot{q} = \kappa \nabla T\)
Measuring $\kappa$ in dusty plasmas: Setup

Experiment setup

Nosenko et al., PRL (2008)
Measuring $\kappa$ in dusty plasma
Measuring $\kappa$ in dusty plasma

Outside laser-heated region, the temperature is about exponential decay.

$$\frac{dT}{dy} \propto \exp\left(\frac{y}{L_{\text{heat}}}\right)$$
Measuring $\kappa$ in dusty plasma

Model the dusty plasma suspension to continuum, using fluid equation

$$c n v g \nabla T k_B = \nabla g(\kappa \nabla T) - 2v n (T - T^0) k_B + S_{viscous}$$

$$\nabla g(\kappa \nabla T) = 2v n (T - T^0) k_B$$

$$d T / dy \propto \exp( y / L_{heat} )$$

$$\kappa = 2v n L_{heat}^2 k_B$$

Thermal diffusivity $\chi$ is

$$\chi = \kappa / c n = 2v L_{heat}^2 k_B / c = 2v L_{heat}^2 k_B$$
Thermal conductivity determination in our experiment

We use $K T_y$ as the kinetic temperature to determine $\kappa$. 

$$\langle v_{\text{thermal } y}^2 \rangle$$ 

$$\frac{d \langle v_{\text{thermal } y}^2 \rangle}{dY}$$
Thermal conductivity determination in our experiment

Thermal diffusivity

\[ \chi = \kappa / cn = 2 \nu L^2_{\text{heat}} \]

Based on our fitting results:

\[ L_{\text{heat}} = 4.45 \text{ bins} \]

The determined thermal diffusivity is

\[ \chi = \kappa / cn = 2 \nu L^2_{\text{heat}} = 7.4 \text{ mm}^2 / \text{s} \]
Equation of energy exchange of fluids

For 2D fluids which has a symmetry in the $x$ direction, the energy equation is:

$$T \frac{DS}{Dt} = \frac{1}{2} \frac{\eta}{\rho} \left( \frac{\partial v_x}{\partial y} \right)^2 + \frac{\kappa}{\rho} \frac{\partial^2}{\partial y^2} (T)$$

energy dissipation per unit mass

**Batchelor, An Introduction to Fluid Dynamics (1979)**

viscous heat  heat conduction

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For our dusty plasma, we also need to consider:

- The dissipation due to gas damping
- The energy input due to laser manipulation
Dissipation due to gas damping

Gas drag friction coefficient
\[ \nu = \frac{F}{m_{\text{dust}}} \nu_{\text{dust}} = 2.7 \, \text{s}^{-1} \]

Gas damping force
\[ F = \nu m_{\text{dust}} \nu_{\text{dust}} \]

Energy dissipation due to gas damping per unit mass
\[ P = F \nu_{\text{dust}} / m_{\text{dust}} = \nu \nu_{\text{dust}}^2 \]
Hydrodynamic Equations

Navier-Stokes equation

momentum

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} = - \frac{\nabla p}{\rho} + \frac{1}{\rho} \nabla g(\eta \nabla v) + \left[ \frac{\xi}{\rho} + \frac{\eta}{3\rho} \right] \nabla(\nabla g v)
\]

energy

\[
\frac{D H}{Dt} = \frac{1}{\rho} \frac{D p}{Dt} + \frac{1}{\rho} \nabla g(\kappa \nabla T) + \frac{1}{\rho} \Phi
\]

energy dissipation per unit mass

Batchelor, An Introduction to Fluid Dynamics (1979)
Forces in the momentum equation

The main horizontal forces acting on the dust particles:

- Inter-particle electric repulsion (Coulomb collisions)
  - Responsible for transport coefficients \( \eta \) and \( \kappa \)
- Radiation pressure force from laser manipulation
- Gas drag (friction) force
Profile of kinetic energy (contribution due to y-component of particle velocity)

with the contribution of the flow velocity

\[
2k_B T / m_{\text{part}} = \frac{1}{N} \sum_{i=1}^{N} [v_{\text{part}i} - v_{\text{flow}(y)}]^2
\]

\(v_{\text{flow}(y)}\) is flow velocity at \(y\), interpolated between grid centers

without the contribution of the flow velocity
What is thermal conductivity?

Thermal conductivity ($\kappa$)

The property of a material’s ability to conduct heat.

Heat flux $\dot{q} = \kappa \nabla T$
What is viscosity?

### Viscosity ($\eta$)

A measure of how a fluid flows under a shear force (shear stress).

$$\eta = \frac{\text{shear stress}}{\frac{\partial v_x}{\partial y}}$$

Example:
Couette flow, apply a stress (force per unit area) to the top plate:
- Moving boundary plate
- Stationary boundary plate
- Pull upper plate with shear stress = force/area