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Factor intensity is a two-dimensional concept with no clear meaning when there are numerous factors of production and numerous outputs. The present article considers the potential application of mean weighted factor intensity, a cardinal ranking across products for each factor of production. If output is measured as valued added, mean weighted intensity also ranks factors for each product. Mean weighted factor intensity successfully anticipates the comparative static elasticities of a high dimensional factor proportions model of production and trade, and may prove useful in predicting long term export potential.

I. AN EMPIRICAL MEASURE OF FACTOR INTENSITY FOR MANY FACTORS AND PRODUCTS

A shortcoming of the factor proportions theory of production and trade is that the fundamental concept of factor intensity cannot be directly applied to data with various factors of production and numerous products. Factors can include labor skill groups, natural resources, energy inputs, and capital vintages or...

The present note defines factor intensity relative to the mean of each input across industries, generating a bilateral ranking for each factor and product. While there are no necessary links between mean weighted factor intensity and theoretical properties of production models, the metric may prove a useful empirical guide. It successfully anticipates the comparative static properties of an applied general equilibrium model of production and trade in the literature. Factor winners and losers due to trade policy may be anticipated with a relatively straightforward examination of factor intensities without having to estimate production functions or rely on model simulations under various industrial structures.

II. FACTOR INTENSITY BASICS

Let $a_{ij}$ be the input of factor $i$ per unit of product $j$. With two factors and two products the ratio of inputs across products generates the intensity ranking $a_{11}/a_{21} > a_{12}/a_{22}$. This two-dimensional measure can be extended to any number of products. In the $2 \times 3$ model, the intensity ranking is $a_{11}/a_{21} > a_{12}/a_{22} > a_{13}/a_{23}$ with industry 1 using factor 1 intensively, industry 3 using factor 2 intensively, and industry 2 in the middle. The $2 \times n$ small open economy is over-determined but assumptions can be relaxed to create a tractable model.
In the $2 \times 2$ model, factor 1 has a higher opportunity cost in product 1 in the converted intensity condition $a_{11}/a_{12} > a_{21}/a_{22}$ and adding factors extends this opportunity cost ranking. In the $3 \times 2$ model, $a_{11}/a_{12} > a_{21}/a_{22} > a_{31}/a_{32}$ and factor 1 is intensive in industry 1, factor 3 in industry 2, and factor 2 in the middle. Whether a country exports the product using its most abundant factor most intensively depends on factor intensity as well as substitution as developed by Ruffin (1981), Jones and Easton (1983), and Thompson (1985).

With as few as three factors and three products, there is no factor intensity ranking. In the $3 \times 3$ model, industry 1 might use factor 1 most intensively relative to industry 2 but least intensively relative to industry 3. The proposed mean weighted factor intensity MWFI provides a ranking for high dimensional models.

III. MWFI

The mean weighted factor input factor $i$ in product $j$ is as $\mu_{ij} = a_{ij}/m_i$ where $m_i = \sum_j a_{ij}/n$ is the mean input of factor $i$ across the $n$ products. Comparing this intensity across products with the same units of output (tons for instance) is straightforward but the typical data involves outputs with different physical units. Following applied production analysis, define a unit of output as the amount worth one unit of numeraire. Output is then value added and the mean weighted factor intensity MWFI can be compared across products. If $\mu_{ij} > 1 > \mu_{ih}$ industry $j$ uses factor $i$ more intensively than the average industry, and industry $h$ less intensively. The ratio $\mu_{ij}/\mu_{ih} = a_{ij}/a_{ih}$ indicates the opportunity cost of product $j$ in terms of product $h$. If $\mu_{ij}/\mu_{ih} > \mu_{kj}/\mu_{kh}$ industry $j$ has a higher opportunity cost than industry $h$ in factor $i$ relative to factor $k$.

Comparing factors across an industry $\mu_{ij} > 1 > \mu_{kj}$ implies industry $j$ uses factor $i$ intensively relative to its average input and factor $k$ less intensively. Comparison across factors rescales the underlying factor intensity ranking $\mu_{ij}/\mu_{kj} = (a_{ij}/a_{kj})(m_i/m_k)$. If $\mu_{ij}/\mu_{kj} > \mu_{ih}/\mu_{kh}$ factor $i$ is intensive in industry $j$ relative to factor $k$ in industry $h$. 

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Rescaling unit inputs has no effect on the comparative static effects of changing prices and endowments on factor prices and outputs in the general equilibrium production model. The total endowment \( v_i \) of factor \( i \) would be rescaled to \( v_i/\mu_i \) and factor prices \( w_i \) rescaled to \( \mu_i w_i \). Competitive pricing conditions \( p_j = \Sigma a_{ij} w_i \) remain consistent with exogenous world prices \( p_j \). Full employment conditions \( v_i = \Sigma a_{ij} x_j \) include rescaled endowments and unit inputs. Factor shares, industry shares, and substitution elasticities are unaffected by the rescaling.

In even models with the same number of factors and products, outputs are uniquely determined given factor endowments. In uneven models such as the specific factors model, the pattern of production is not determined by endowments. Nevertheless, the present MWFI may prove useful in data exploration and applications.

Collect the mean weighted factor inputs with \( r \) factors and \( n \) products into the factor intensity matrix \( F_{rxn} \). The following section examines how well \( F_{rxn} \) anticipates the comparative static results of an applied factor proportions model. Suppose there are \( c \) countries and define matrix \( B_{nxc} \) as the mean weighted factor abundance matrix. An empirical test of factor content theory would involve the empirical relationship between \( F_{rxn}, B_{nxc}, \) and \( X_{nxc} \) matrix of net exports across countries.

IV. AN APPLICATION OF MWFI

The \( 9 \times 3 \) model of the U.S. economy in Thompson (1990) provides a glimpse into potential application of MWFI. The first columns in Table I for the three sectors agriculture \( A \), manufacturing \( M \), and services \( S \) are the derived factor shares \( \theta_{ij} = a_{ij} w_i / p_j \) for capital and eight Census skill groups of labor. Output is value added with a unit of output defined by \( p_j = 1 \). Capital shares are residuals of value added after labor shares. Mean weighted factor shares equal mean weighted factor intensities since \( 30_{ij}/\Sigma_j \theta_{ij} = 3a_{ij} w_i / \Sigma_j a_{ij} w_i = 3a_{ij}/\Sigma_j a_{ij} = \mu_{ij} \).
The largest labor factor shares are operators in manufacturing at .286 and professionals in services at .269. Other large labor shares are technical/sales labor in services .211, crafts in manufacturing .167, and resource labor in agriculture .139. The large residual capital share in agriculture .576 implicitly includes land input.

Each input has two sets of ratios across the three sectors. Input ratios in services relative to manufactures are \( S/M = \theta_{iS}/\theta_{iM} \) with the skilled wage \( w_i \) equal across sectors and \( p_A = p_M = 1 \). This input ratio \( S/M \) and the service/agriculture intensity \( S/A \) are the last two columns for each factor in Table I.

Figure 1 shows the service/manufacturing \( S/M \) and service/agriculture \( S/A \) rankings. Service labor, technical labor, and professional labor are the most intensive inputs in services relative to both other sectors but beyond those three inputs the rankings are not similar. Among the other inputs, capital is used intensively in services relative to manufactures but not relative to agriculture. The opposite is true for operators. For all inputs the correlation between the two rankings is 0.07 and excluding the three intensive inputs the correlation is -0.40.
MWFI $\alpha_{ij}$ is reported in the second column of Table I for each sector. Reading down columns compares factors for the sector and yields the same ordinal ranking as factor shares. Agriculture uses capital more intensively than any type of labor except resource labor. Manufactures use operators most intensively. Services use service labor and technical/sales labor. Figures 2–4 present the rankings in Table I.
Capital is more than twice as intensive in agriculture as in the other two sectors. Resource labor is virtually specific to agriculture. The service sector uses professional labor about twice as intensively as manufacturing which uses it about twice as
intensively as agriculture. The service sector uses technical and service labor more intensively than the other two sectors. Manufacturing uses crafts, operators, and handlers intensively.

Operators are about four times as intensive in manufacturing relative to services and seven times as intensive relative to agriculture. Transport, technical, and professional labor all have about average intensity in manufacturing, which uses every type of labor except service and resource labor more intensively than capital. The service sector uses service, technical, and professional labor the most intensively. Transport labor has the least intensity variation and is close to average intensity. The simple intensity scaling illustrates the potential usefulness of the mean intensity measure.

Factor shares are a misleading guide to factor intensity in this example. Reading down the $\theta_{im}$ column for manufacturing, capital appears more intensive than handlers but reading down the mean intensity $\alpha_{im}$ column capital is used only 42% (0.61/1.56) as intensively as handlers. There are other such examples.

The MWFI anticipate the Stolper-Samuelson (1941) $dw_i/dp_j$ elasticities reported in the third columns for each sector in Table I. The model uses estimates of translog production functions across states but these $dw_i/dp_j$ elasticities are insensitive to a wide range of factor substitution depending almost entirely on factor shares. An increase of 1% in the price of manufactures raises the operator wage over 3% and lowers the service worker wage by just under 1%. Figures 2–4 also present these $dw/dp$ elasticities.

The correlation of the comparative static $dw/dp$ elasticity vector with factor shares across all three sectors is only 0.32 while its correlation with mean weighted factor intensities is 0.74. The correlation between factor shares and mean weighted factor intensities of 0.42 indicates the difference in the two measures. These properties are apparent in Figures 2–4.

Due to reciprocity in the comparative static results, mean weighted factor intensities also anticipate the effects of changing
factor supplies on outputs. Countries more abundant in a factor are expected to produce and export more of the products using that factor intensively, at least given identical homothetic preferences and no transport costs. With many factors and countries, a similar mean weighted measure of factor abundance can be formulated. Whether the MWFI and abundance anticipate the direction of trade is an empirical issue.

IV. CONCLUSION

Mean weighted factor intensities provide a metric to anticipate and interpret general equilibrium properties of high dimensional factor proportions models of production and trade. They are comparable across products for each factor and across factors for each product. In contrast, factor shares are comparable only across factors for each product and are apparently misleading guides to theoretical predictions.

Examination of the empirical links between mean weighted factor intensities and trade would provide a test of the relevance of factor content theory. Mean weighted factor abundance matrices can be used alongside mean weighted factor intensity matrices given data for various factors of production, products, and countries. An empirical test of factor content theory would involve the empirical relationship between the factor abundance matrix, the factor intensity matrices, and the matrix of net exports across countries.

As a forward looking application, consider the potential effect of liberalized trade inside the evolving Free Trade Area of the Americas FTAA. The 34 countries in FTAA will increase trade in hundreds of manufactured goods, services, and natural resource products classified with the NAICS system. There is associated input data on labor, energy, and residual capital, and there are some data for skilled labor groups. A comparison of mean weighted intensities across industries and mean weighted abundance across
countries would predict which countries will export which products. Countries with above average abundance in a factor might be expected to export products with above average intensity in that factor. Labor groups with below average abundance could expect falling wages. Such projections would avoid estimation of cost or production functions in detailed production models. Policymakers could use the projections to help make a decision about whether to invest in a container port at a particular location, or whether to alter tax rates in anticipation of the income redistribution that will follow trade.

REFERENCES


