Duopoly quotas and relative import quality

T. Randolph Beard\textsuperscript{a}, Henry Thompson\textsuperscript{b,\,*}

\textsuperscript{a}Department of Economics, Auburn University, Auburn, AL 36849, USA
\textsuperscript{b}Department of Agricultural Economics, Auburn University, Auburn, AL 36849, USA

Received 12 December 2000; received in revised form 15 December 2001; accepted 3 January 2002

Abstract

The present paper develops a two-stage game of quality and quantity competition for an international duopoly in the home market with linear demand and cost functions. While a quota increases domestic output, domestic quality falls to the monopoly level. Import quality may rise or fall with the quota, and the upgrading noted in the empirical literature may occur only relative to home quality.

\textsuperscript{©} 2002 Elsevier Science Inc. All rights reserved.

\textit{JEL classification}: F0; F1
\textit{Keywords}: International; Duopoly; Import; Quality

1. Introduction

Quotas are widely believed to create foreign quality upgrading based on theory and empirical evidence. Theoretical studies include Das and Donnenfeld (1989), Falvey (1979), Mayer (1982), Rodriguez (1979), and Santoni and VanCott (1980), and empirical studies include Anderson (1985), Aw and Roberts (1986), Feenstra (1984), MacPhee (1974), Meier (1973), and Patterson (1966). The present paper develops a simple general model of an international duopoly in which a quota may lead to absolute import quality downgrading but relative upgrading.
Krishna (1987) points out that a quota on a foreign monopolist would lead to import downgrading if the value of quality for the marginal consumer is less than for the average consumer. Chen (1992) makes the point that quality downgrading could be a signal to sustain oligopoly collusion. Ries (1993) shows that a relatively low quality foreign Cournot competitor will not upgrade with a quota.

Sweeney, Thompson, and Beard (1996) use a Mussa and Rosen (1978) demand specification to compare models of simultaneous and sequential quality choice, uncovering a range of possible quality responses to a quota, and Das and Donnenfeld (1987) utilize a similar demand structure. While such a restrictive interpretation of quality as a vertical characteristic might be sensible in some contexts, the present paper uses a more general specification without an implied ordinal quality ranking.

In the present model of an international duopoly competing in the home market, linear demand facing each firm responds to its own quality as well as output and quality of the other firm. The two-stage game has quality-differentiated products. The two firms select product qualities in a noncooperative stable Nash equilibrium, then engage in Cournot quantity competition in the second stage. Information is complete in the one-shot game. Cost is linear and quality raises marginal cost. With a quota, the home firm reduces quality to its monopoly level. The results are consistent with observed import quality upgrading but perhaps only relative to home quality.

2. An international duopoly without a quota

Consider the inverse demand function facing the home firm (Eq. (1)),

\[ p = p(Q, Q^*, q, q^*), \]  

(1)

where \( p \) represents price, \( Q \) quantity, \( q \) quality, and \( * \) foreign variables. Signs of first derivatives are \( \delta p/\delta Q = p_Q < 0 \), \( p_{Q^*} < 0 \), \( p_q > 0 \), and \( p_{q^*} < 0 \). Home and foreign goods are substitutes, reflected by the negative \( p_{Q^*} \). Higher quality increases own demand and lowers competitor’s demand. Assume linear demand \( p_{QQ} = 0 \) with no second-order effects, \( p_{ii} = p_{ij} = 0 \), for \( i, j = Q, Q^*, q, q^* \). Linear inverse demand for the foreign firm in the home market \( p^* = p^*(Q, Q^*, q, q^*) \) has symmetric properties.

Both quantity and quality are assumed to have linear marginal costs with a positive crosseeffect. The cost function of the home firm is written as (Eq. (2))

\[ c = c(Q, q), \quad c_Q > 0, c_q > 0, c_{QQ} = c_{qq} = 0, c_{qQ} = c_{Qq} > 0. \]  

(2)

Increased quality raises marginal cost. The cost function of the foreign firm \( c^* = c^*(Q^*, q^*) \) has symmetric properties.

In the second stage, quality is fixed and both firms choose quantity to optimize profit. In the unconstrained model with no quota, home firm profit is (Eq. (3))

\[ \pi = [p(Q, Q^*, q, q^*) - c(Q, q)]Q. \]  

(3)
The first-order condition for profit maximization with respect to output is

\[ 0 = \pi_Q = \frac{\delta \pi}{\delta Q} = p(\cdot) - c(\cdot) + (p_Q - c_Q)Q = p - c + \varphi Q, \tag{4} \]

where \( \varphi = p_Q - c_Q < 0 \). Optimal output is \( Q = (c - p)/\varphi \) and positive output implies \( p > c \) and \( \pi > 0 \). The home firm responds to foreign output according to

\[ \frac{\delta Q}{\delta Q^*} = \frac{Q_Q^*}{p_Q} = -\frac{p_Q^*}{\varphi < 0}, \tag{5} \]

the slope of the linear home reaction function in the second stage. Symmetrically, the foreign firm’s reaction function is

\[ \frac{\delta Q^*}{\delta Q} = \frac{Q_Q^*}{p_Q^*} = -\frac{p_Q^*}{\varphi^* < 0}, \tag{6} \]

where \( \varphi^* = p_Q^* - c_Q^* < 0 \). A stable Nash equilibrium requires a steeper reaction function for the foreign firm as in Fig. 1. The equilibrium outputs are \( Q_e \) and \( Q_{e^*} \). Invert the foreign firm’s reaction function in Eq. (6) to find its slope relative to the home reaction function, \( -\frac{p_Q^*}{\varphi^*/p_Q^*} \), implying the stability condition

\[ p_Q^*p_Q^* < \varphi^*. \tag{7} \]

To link first-stage quality to quantity, fully differentiate the first-order condition in Eq. (4) to find

\[ 2\varphi dQ + p_Q^*dQ^* + \psi dq + p_q^*dq^* = 0, \tag{8} \]
where $\psi \equiv p_q - c_q - c_{Oq} Q$, which may be positive or negative. Combine Eq. (8) with the symmetric condition for the foreign firm into the comparative static system,

$$
\begin{pmatrix}
2\varphi & p_{Q*} \\
p_q & 2\varphi
\end{pmatrix}
\begin{pmatrix}
dQ \\
dQ*
\end{pmatrix}
= -
\begin{pmatrix}
\psi & p_{q}^* \\
p_q^* & \psi^*
\end{pmatrix}
\begin{pmatrix}
dq \\
dq^*
\end{pmatrix},
$$

(9)

where $\psi^* \equiv p_{q}^* - c_{q}^* - c_{Oq}^* Q^*$. Solving for the effects of quality on quantity in Eq. (9),

$$
\frac{\delta Q}{\delta q} \equiv Q_q = \frac{(p_{Q^*}p_q^* - 2\varphi^*\psi)}{\Delta}
$$

(10)

and

$$
Q_q^* = \frac{(p_{Q^*}p_q^* - 2\varphi p_q^*)}{\Delta}
$$

with determinant $\Delta \equiv 4\varphi \varphi^* - p_{Q^*}p_q^*$. The stability condition in Eq. (7) implies $\delta > 0$, but none of the results in Eq. (10) can be signed, even the own effects of quality on quantity.

When quality changes, there are three effects on quantity. If $q$ increases in the first stage for a given $q^*$, marginal cost increases, which would lower output. Higher quality also raises marginal revenue, which would raise $Q$. Finally, the strategic effect of an increase in $q$ is felt on marginal revenue through adjustment of $Q^*$ in Eq. (10). The ambiguous effects occur even in the simple case of linear demand and cost.

**Result 1:** Higher domestic quality raises marginal cost and marginal revenue for the domestic firm and has a strategic effect on foreign output. The net effect on domestic output is ambiguous.

To consider the potential of quality crosseffects on demand, assume for the moment there are none: $p_{q^*} = p_q^* = 0$. The comparative static effects of a change in domestic quality in Eq. (10) then simplify to Eq. (11)

$$
Q_q = -2\varphi^*\psi/\Delta \text{ and } Q_q^* = \psi p_q^*/\Delta,
$$

(11)

with $\text{sgn} Q_q = - \text{sgn} Q_q^*$. The implication is that $Q_q$ and $Q_q^*$ could have the same sign only with quality crosseffects. A similar result holds for $Q_q^*$ and $Q_q$. 

**Result 2:** A change in one firm’s quality can affect home output and foreign output in the same direction only if there are quality crosseffects on demand.
In the first stage, both firms are assumed to be aware of the second-stage solution. The home firm optimizes profit with respect to quality, taking into account the second-stage effects of its quality on home and foreign quantities,

\[ 0 = \frac{\delta \pi}{\delta q} \equiv \pi_q = (\psi + p_{Q^*}Q_{q^*})Q + (p - c + \varphi Q)Q_q, \quad (12) \]

where \( Q_q \) and \( Q_{q^*} \) are second-stage comparative static solutions in Eq. (10), and \( Q \) is the optimal second-stage output. Firms optimize with their choice of quality both directly and indirectly through second-stage effects. Using the first-order condition in Eq. (4) from the second stage, the first-order condition in Eq. (12) simplifies to

\[ 0 = (\psi + p_{Q^*}Q_{q^*})Q. \quad (13) \]

Substitute \( Q_{q^*} \) from Eq. (10) into Eq. (13) and solve for \( \psi \) to find (Eq. (14))

\[ \psi = p_{q^*}p_{Q^*}/2<0. \quad (14) \]

Symmetrically, \( \psi^* = p_{q^*}p_{Q^*}/2<0 \). None of the comparative static effects of quality on quantity in Eq. (10) can be signed.

A home monopolist would provide quality to optimize profit where \( \psi = 0 \). Whether the home duopolist would provide more or less quality depends on the sign of \( Q_q \) since \( \delta \psi/\delta q \equiv \psi_q = -c_{Qq}Q_q \). Increased quality of either firm can be associated with any set of quantity changes, but more definite results are found in the presence of a binding quota.

3. Duopoly with a binding quota constraint

The second stage constrained Nash equilibrium in the presence of a binding quota is also illustrated in Fig. 1. The quota \( Q_b^* \) is set at a level less than the unconstrained Nash equilibrium quantity \( Q_e^* \) given optimal qualities. Domestic output is higher than it would be in the unconstrained model set by the first-order condition Eq. (5). In the second-stage Nash quantity competition, there is no quantity response by the foreign firm due to the binding quota. The foreign firm cannot increase output and would not lower it.

The first-order condition for the home firm with respect to output is the same as for the unconstrained models, namely Eqs. (4) and (8). For the foreign firm, there is no profit-maximizing adjustment in \( Q^* \), and the comparative static effect of quality on quantity reduces to the first equation in Eq. (9). The set of second-stage quantity adjustments in the presence of a binding quota analogous to Eq. (10) are

\[ Q_q = -\psi/2<0 \]

\[ Q_{q^*} = -p_{q^*}/2<0, \quad (15) \]

\[ Q_{Q^*} = -p_{Q^*}/2<0. \]
Result 3: In the presence of a binding quota, an increase in foreign quality lowers home output due to a decline in marginal revenue. A tighter quota raises domestic output.

The relationship between home quality and quantity depends on the sign of $\psi$. Turning to the first stage, the first-order condition in quality reduces to (Eq. (16))

$$0 = \pi_q = \psi Q,$$

since $Q^*_q = 0$ in Eq. (13). The implication is that $\psi = 0$, the first-order condition for a monopolist. In addition, $\psi_q = -c_{qq}Q_q = 0$ from Eq. (15) given $\psi = Q_q = 0$.

Result 4: A binding quota allows the home firm to produce the monopoly level of quality, the lack of quantity competition eliminating quality competition. Optimal output of the home firm does not depend on home quality.

For the foreign firm, the first-order condition for profit maximization with respect to quality under a binding quota constraint is (Eq. (17))

$$0 = \pi^*_q = (p^*_q Q^*_q + p^*_q - c^*_q)Q^*,$$

implying $p^*_q - c^*_q = -p^*_q Q^*_q$. Substituting for $Q^*_q$ from Eq. (15) yields Eq. (18)

$$p^*_q - c^*_q = p_q^* p^*_q / 2 \varphi < 0.$$  

implying foreign quality is provided above the monopoly level where $p^*_q$ would equal $c^*_q$.

Result 5: A tighter quota causes both home and foreign quantities to fall. Home quality remains at the monopolistic level. Foreign quality may rise or fall.

If there were equal home and foreign quality in the unconstrained equilibrium, a quota would increase the relative quality of the foreign product $q^*/q$ because $q^*$ remains above its monopoly level while $q$ falls to its monopoly level. If there were higher domestic quality, the quota would raise relative import quality. Even if $q^*$ were to fall, $q^*/q$ would rise due to the reduction of $q$ to its monopoly level. If there were higher foreign quality, the quota might lower the relative quality of the foreign product.

Result 6: A quota must raise relative import quality if import quality is no less than home quality. If import quality is higher than domestic quality, a quota may lower relative import quality.

A negative crosseffect in foreign demand between output and quality $p^*_q Q^*_q < 0$ would favor import quality upgrading, the reduction in foreign quantity raising the marginal revenue of foreign quality. Similarly, a positive $p^*_q Q^*_q$ and a negative $p^*_q q^*$ would favor foreign quality upgrading and an increase in relative import quality. Opposite second-order effects would favor import quality downgrading.
4. Conclusion

A quota on a foreign duopoly competitor affects the quality of both imports and the competing home product in the present general model of an international duopoly and quality. A quota protects the domestic firm from having to compete on the quantity margin, and in response, it reduces quality. Import quality may decline relative to what it would have been with duopoly competition, but domestic quality falls even farther. The observed import quality upgrading may only reflect relative import quality.

References


