KEIO ECONOMIC STUDIES **54**, 73–80 (2018)

PARTIAL SPECIALIZATION WITH CONSTANT COST COMPARATIVE ADVANTAGE

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First version received November 2018; final version accepted March 2019

Abstract: In the model of constant cost production and trade, each country completely specializes to export its own comparative advantage good. The present paper extends this theory to allow partial specialization motivated by trade with a country too small to support complete specialization. A partially specialized large country gains from trade at international prices. With three or more countries and goods, partial specialization introduces the potential of multiple exports. The pattern of trade becomes much more realistic introducing partial specialization and trade at international prices.

Key words: Comparative advantage, specialization, trade.
JEL Classification Number: F10.

Two countries completely specialize according to comparative advantage given the constant cost factor mix of Ricardo (1817) given the implicit assumption that each can produce enough for the other to gain from trade. The constant cost trade model of Jones (1961), Chipman (1965), Ethier (1986), and Ruffin (2013) assumes complete specialization. The present paper makes the point that partial specialization can lead to trade with a small country that cannot support complete specialization.

With partial specialization, the economy produces at an interior point along its linear production frontier implying autarky prices remain unchanged with trade. While domestic prices are unaffected, trade takes place at international prices. The terms of trade allow a large country to gain from trade consuming beyond its production frontier. Partial specialization also introduces the potential of optimal tariffs to improve the terms of trade.

Partial specialization generalizes to trade among three or more countries and goods. Constant cost trade with three countries and goods is developed by Bastable (1903), Viner (1932), and Graham (1948). The present partial specialization introduces the potential of trade with one or two small countries as well as the export of more than a single good.

The model with many countries and goods starts with the Jones (1961) efficiency

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applying the activity analysis of McKenzie (1954). Each country completely specializes to export its efficient good to all other countries assuming country sizes provide sufficient export for all other countries. The present model allows partially specialized large countries to gain from trade and introduces the potential to export of more than the single efficient good. The trade pattern with partial specialization becomes more complex.

The first section develops the 2×2 model with two countries and two goods introducing partial specialization and trade with a small country as well as an optimal tariff. The second section develops the 3×3 model allowing multiple exports, and discusses the high dimensional model with many countries and goods.

1. PARTIAL SPECIALIZATION IN 2×2 PRODUCTION AND TRADE

Constant unit input coefficients for countries A and B in goods 1 and 2 are written,

$$\begin{pmatrix} a_{1A} & a_{1B} \\ a_{2A} & a_{2B} \end{pmatrix}. \tag{1}$$

Country A has the comparative advantage in good 1 according to,

$$A_{12} \equiv a_{1A}/a_{2A} B_{12} \equiv a_{1B}/a_{2B}\,. \tag{2}$$

Competitive pricing implies the price p_{hk} of good h in country k and its factor price f_k are related by input coefficients according to $p_{hk} = f_k a_{hk}$. Let F_k be the factor endowment of country k. Complete specialization implies either $x_{1A} = F_A/a_{1A}$ or $x_{2B} = F_B/a_{2B}$ at the endpoints of the linear production frontier.

The assumption in the literature is that each completely specializes with its endowment providing sufficient export for the other to gain from trade. The terms of trade can be written $tt \equiv p_1^*/p_2^*$ where p_h^* is the international price of good h. The relative price condition for trade to take place is $A_{12} < tt < B_{12}$ with the terms of trade flanked by relative autarky prices in (2). The international equilibrium $tt$and trade levels are determined by utility maximization in the two trading countries. For simplicity, the present paper assumes homothetic identical preferences to focus on production short of an explicit deterministic solution.

Figure 1 illustrates small country B unable to produce enough good 2 for country A to completely specialize. The two international prices with trade must be ranked between autarky prices according to $p_{1B} > p_1^* > p_{1A}$ and $p_{2A} > p_2^* > p_{2B}$. Small country B completely specializes and trades to point T_B with gains from complete specialization at $tt = p_1^*/p_2^*$. Large country A partially specializes to point b and trades at the terms of trade tt. Both countries implicitly maximize utility.

Autarky prices p_{hA} in large country A are maintained with partial specialization along the production frontier. Homothetic preferences and the unaffected domestic prices imply country A consumes the two goods in the same ratio simplifying the analysis. Consumption at point T_A is determined by maximizing utility along tt subject to

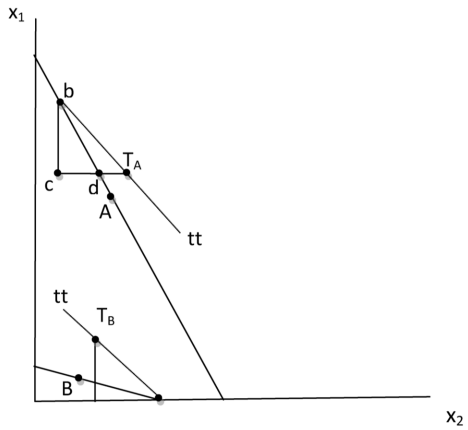

Figure 1. Partial specialization and trade with a small country

domestic prices p_{hA}. Small country B produces and maximizes utility according to tt. Both countries gain from trade with utility higher than autarky points A and B.

The value of autarky production in country A is $Y_A = f_A(a_{1A}x_{1A} + a_{2A}x_{2A}) = f_A F_A$. Trade is balanced on the trade triangle bcT_A at the international prices p_h^* according to $X_A = p_1^*(x_{1A}^* - c_{1A}^*) = M_A = p_2^*(c_{2A}^* - x_{2A}^*)$ where $X_A = M_B$ and $M_A = X_B$ and X_j is export revenue and M_j import spending in country j. Production of good 1 by country A must cover consumption in both countries according to $x_{1A}^* = c_{1A}^* + c_{1B}^*$ implying $x_{2A}^* + x_{2B}^* = c_{2A}^* + c_{2B}^*$ as both countries produce and consume good 2.

The value of consumption in large country A with trade at world prices $C_A^* = p_1^* c_{1A}^* + p_2^* c_{2A}^*$ equals the value of output $Y_A^* = p_1^* x_{1A}^* + p_2^* x_{2A}^*$ while the value of consumption in autarky is $C_A = p_{1A}c_{1A} + p_{2A}c_{2A}$. Trade increases income in country A according to $Y_A^* = C_A^* = f_A^* F_A < Y_A = C_A = f_A F_A$ with the implied increase in the factor price $f_A^* > f_A$ reflecting the gains from trade.

Country A has a trade deficit evaluated at domestic autarky prices according to $p_{1A}(x_{1A}^* - c_{1A}^*) < p_{2A}(c_{2A}^* - x_{2A}^*)$ although trade is balanced at international prices p_h^*. The increase in income can be expressed in terms of international prices and export according to $Y_A^* - Y_A = (tt - a_{1A}/a_{2A})(x_{1A}^* - c_{1A}^*)/p_2^* = (f_A^* - f_A)F_A > 0$ as $tt > a_{1A}/a_{2A}$ and $x_{1A}^* > c_{1A}^*$. This increase in income in terms of imported good 2 is the line segment cT_A in Figure 1 along the bottom of the trade triangle. The difference bT_A between the value of export bc and its value cd inside the country is equal to the increase in income.

To illustrate the importance of country size to trade, start with two large countries and consider an increasingly smaller F_B falling to the level F_B^{min} that leaves country A indifferent to trade with the international relative price falling to the domestic level a_{1A}/a_{2A}. If F_B falls below F_B^{min} complete specialization and trade collapse. The present paper

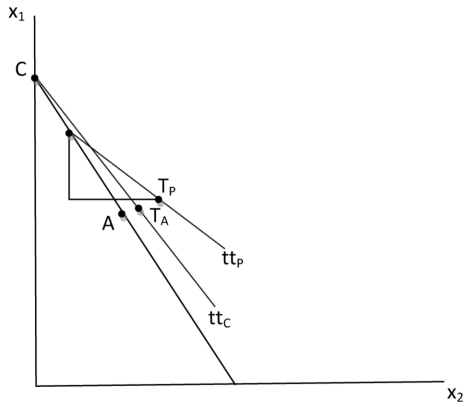

Figure 2. Partial specialization and improved terms of trade

makes the point that country A can partially specialize and gain from trade. The minimum size F_B^{MIN} is required to support partial specialization as trade has to be able to move consumption in country A beyond its production frontier. If F_B falls below F_B^{MIN} partial specialization and trade collapse.

Figure 2 illustrates the gains from an optimal tariff that improves the terms of trade enough to offset the loss due to the decrease in specialization. Utility is maximized in the two countries although Figure 2 focuses on a single country and does not explicitly include indifference curves. The gains from trade are due to better terms of trade tt_P with partial specialization at point P compared to complete specialization at point C and the resulting terms of trade tt_C. The full deterministic equilibrium includes both countries and utility functions.

The restricted level of the export with the optimal tariff raises its relative price on the international market in Figure 2. An equivalent quota on exports or imports would have the same effect on the trade pattern. The economy moves from complete specialization at point C to partially specialized production point P as the relative price of imported good 2 falls from tt_C to tt_P in the full equilibrium including both countries. The optimal tariff is found with the highest utility level between autarky production point A and complete specialization. Figure 2 is analogous to the optimal tariff in the neoclassical model except that domestic prices are unaffected short of complete specialization.

2. PARTIAL SPECIALIZATION IN 3 × 3 PRODUCTION AND TRADE

Adding country C and good 3 to the two dimensional input matrix (1) leads to,

$$\begin{pmatrix} a_{1A} & a_{1B} & a_{1C} \\ a_{2A} & a_{2B} & a_{2C} \\ a_{3A} & a_{3B} & a_{3C} \end{pmatrix}. \tag{3}$$

Thompson (2001) explores the possible patterns of specialization and trade among

three countries in three goods. The Jones (1961) efficiency is identified by the minimum of the six cross products,

$$\begin{array}{ll} (a) & a_{1A}a_{2B}a_{3C} \\ (b) & a_{2A}a_{3B}a_{1C} \\ (c) & a_{3A}a_{1B}a_{2C} \\ (d) & a_{3A}a_{2B}a_{1C} \\ (e) & a_{1A}a_{3B}a_{2C} \\ (f) & a_{2A}a_{1B}a_{3C}. \end{array} \tag{4}$$

Assume the minimum product is $(4a)$ along the main diagonal. The three inequalities $(4a) < (4d, e, f)$ in the negative direction imply the $A1$-$B2$-$C3$ trade pattern in the relative price comparisons,

$$\begin{array}{ll} (a) & A_{12} < B_{12} \\ (b) & A_{13} < C_{13} \\ (c) & B_{23} < C_{23}. \end{array} \tag{5}$$

'Country A has a lower price of good 1 relative to 2 and country B in $(5a)$ and relative to good 3 and country C in $(5b)$. Similar conditions hold both for country B and good 2 in $(5a)$ and $(5c)$, and for country C and good 3 in $(5b)$ and $(5c)$. Each country has a lower price of its good relative to every other country and their good. The relative price comparisons in (5) are insufficient for comparative advantage, however, as they disregard the two other cross products in (4) in the positive direction.

Comparative advantage requires $(4a) < (4b, c)$ comparing a relative price in a country with a the other two countries trading between themselves. In these conditions,

$$\begin{array}{ll} (a) & A_{12} < C_{13}B_{32} \equiv CB_{12} \\ (b) & A_{13} < B_{12}C_{23} \equiv BC_{13}\,, \end{array} \tag{6}$$

the products of two relative price terms reflect those two countries trading by themselves. For instance CB_{12} is the price of good 1 relative to good 2 between trading countries C and B through their implied common price of good 3. The comparative advantage of country A in good 1 relative to 2 implies it can offer a better price to country B in $(5a)$ and to the trading countries CB in $(6a)$.

Similar conditions hold for country A in good 1 relative to 3 and the trade group BC in $(6b)$ as well as directly relative to country C in $(5b)$. Countries B and C both face similar conditions in their goods relative to the other two trading countries in (6).

Figure 3 illustrates the three dimensional production surface with completely specialized outputs $x_h = F_k/a_{hk}$ at the corners. Edge 0_h represents zero output of good h. In autarky, the country selects its preferred point of production and consumption on the surface. Complete specialization would move output to a corner. Partial specialization moves the country either toward a corner with decreases in both other outputs or toward a side increasing another output as well.

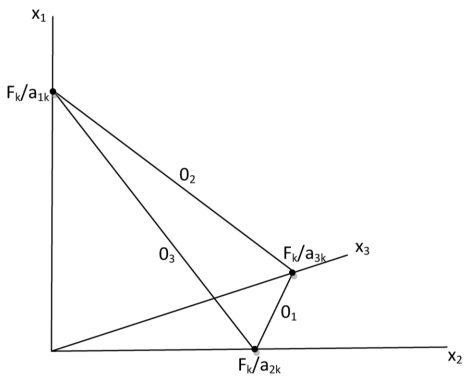

Figure 3. Three factor production frontier

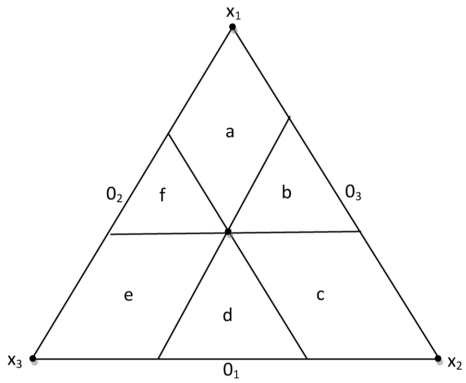

Figure 4. Output adjustments on the three factor production surface

Figure 4 separates output adjustments on the triangular production surface. Assume autarky production and consumption at the intersection of the three lines equidistant from each corner. The line separating region abc from region def is equidistant from corner 3 with lower x_3 in region abc and higher x_3 in region def. The other two lines separate increases from decreases in x_1 and x_2.

The range of possible output adjustments with partial specialization in Figure 4 are illustrated by the directions of output changes in the six regions,

	x_1	x_2	x_3
a	$+$	$-$	$-$
b	$+$	$+$	$-$
c	$-$	$+$	$-$
d	$-$	$+$	$+$
e	$-$	$-$	$+$
f	$+$	$-$	$+$.

(7)

Assume country A is large and country C is small. Country A could move into region f increasing its comparative advantage output x_1 and x_3 as well. Small country C completely specializes in good 3 moving through region e to the x_3 corner in Figure 3. Country B could move into region d increasing its comparative advantage outputx_2 and $x_3$as well due to the limited import from country C. Large countries A and B could completely specialize relative to each other with A moving through region $f$to edge0_2 and B moving through region d to edge 0_1. If country B were small, it would completely specialize moving through region c to the x_2 corner as A maintains all three outputs in region *abf*.

Another possibility is a country exporting a good beyond its comparative advantage. Country A could trade with small country C for good 3 exporting both good 1 and 3 to large country B in exchange for good 2. The present analysis abstracts from country sizes and preferences that would contribute to a deterministic pattern of production and trade.

The implications of partial specialization extend to the higher dimensional models of constant cost production and trade. Comparative advantage in the nxn model starts with comparison of $(n^2 - n)/2$ bilateral relative prices and extends to possible combinations of trading countries ranging from 2 to $n - 1$ members following (4), (5), and (6) in the Jones (1961) efficiency. If country sizes are consistent, complete specialization and trade follow.

Introducing the potential of partial specialization to the $n \times n$ model allows trade with small countries that would be excluded by complete specialization. Partial specialization would generally lead to larger and more efficient countries exporting multiple goods. Partial specialization also introduces optimal tariffs altering the terms of trade and level of trade. The high dimensional pattern of trade would weakly follow comparative advantage in that each country would produce more of its good in the move to free trade.

3. CONCLUSION

Constant cost production leads to trade patterns that depend on country sizes as well as the factor mix. The literature assumes complete specialization implicitly ruling out trade with small countries. The present approach to partial specialization allows gains from trade with small countries that could not support complete specialization. Unlike incomplete specialization, a large country gains from trade due to the higher relative

price of its export. Partial specialization introduces the potential of optimal tariffs as well. With three or more countries and goods, partial specialization introduces multiple exports.

The present principles extend to models with more than a single input. The fixed factor proportions model reviewed by Thompson (2010) features fixed multiple unit inputs leading to partial specialization similar to the factor proportions model. In the missing link model of Ruffin (1988) each factor can produce on its own leading to specialized factors supporting partial specialization.

The potential complexity of trade with partial specialization and trade among many countries in many goods suggests the comparative advantage comparison of Ricardo might go farther than appreciated toward explaining observed trade.

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