Energy in a Physical Production Function:
The US Economy from 1951 to 2008

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A production function motivated by the concept of physical work with energy and labor providing force to interact separately with capital outperforms log linear and translog production functions for US output. Inputs are total energy Btu input, fixed capital assets, and the labor force. Misspecification bias is apparent in estimates without energy input. Compared to labor, energy input has a larger output elasticity and larger price elasticity of marginal cost. Energy is also underpaid with a shadow price three to four times its market price, while labor is overpaid.

Keywords: Energy; production; shadow prices; output elasticity; factor shares

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Energy proves an essential input along in estimates of US gross domestic product with inputs of Btu energy, fixed capital assets, and the labor force. A novel production function motivated by the definition of physical work allows energy and labor to interact separately with capital, outperforming log linear and translog production functions in the present annual data from 1951 to 2008. The misspecification bias in estimates without energy is apparent in the characteristic trends in residuals, generally interpreted as technological change or total factor productivity.

The applied production literature begins with Allen (1938) as reviewed by Takayama (1993). The theory of Ferguson and Pfouts (1962) and Berndt and Christensen (1973) is applied to energy inputs by Cameron and Schwartz (1980), Field and Gerbenstein (1980), Denny, Fuss and Waverman (1981), and Walton (1981). The translog production function of Christensen, Jorgensen, and Lau (1973) focused on energy input by Thompson (2006) has become the foundation of the applied production literature.

Substitution estimates typically uncover evidence of weak energy substitution as in Griffin and Gregory (1976), Burney and Al-Matrouk (1996), Caloghiro, Mourelatos, and Thompson (1997), Barnett, Reutter, and Thompson (1998), Kemfert (1998), Mahmud (2000), Yi (2000), Urga and Walters (2003), and Kuper and van Soest (2003). In contrast, Berndt and Wood (1975) find energy a weak complement with capital.

The present physical production function offers an alternative functional form based on the force provided by labor or energy to produce output as work. The present data pretests suggest estimation with difference or error correction regressions. Energy input improves the estimates with a higher output elasticity than labor and the largest effect on marginal cost. Energy's shadow price is well above its market price, while labor is overpaid.

The first section introduces the proposed physical production function. The second describes the data, followed by sections on log linear and translog estimates. The fifth section presents estimates followed by a section comparing the derived substitution elasticities across models.

1. The physical production function

Physical work equals force times distance, W = FD suggesting the analogy of output Y as work with energy E and labor L providing force. The proposed physical production function is

$$Y = A(LK)^{\alpha_1}(EK)^{\alpha_2} \tag{1}$$

with separate interaction of E and L with K. The effect is to constrain exponents of the log linear production function to $Y = AL^{\alpha_1}E^{\alpha_2}K^{\alpha_1+\alpha_2}$. Euler's theorem holds with constant returns only if $\alpha_1 + \alpha_2 = ½$. Marginal products are

$$Y_K = (\alpha_1 + \alpha_2)Y/K \qquad Y_L = \alpha_1 Y/L \qquad Y_E = \alpha_2 Y/E. \tag{2}$$

Concavity follows if inputs are paid marginal products. For instance, the own energy effect $Y_{EE} = \alpha_1(Y_E E - Y)/L^2$ is negative since $Y = rK + wL + eE$ where r, w, and e are factor prices. Symmetric cross effects such as $Y_{EK} = \alpha_2 Y_K/L = Y_{KE}$ are positive. Inputs are substitutes short of higher order interaction terms, found insignificant in the present application.

The economy minimizes cost by mixing inputs based on input prices. First order conditions of cost minimization for a unit of output lead to the symmetric Hessian matrix of the constrained cost minimization,

$$\begin{pmatrix} 0 & Y_K & Y_L & Y_E \\ . & Y_{KK} & Y_{KL} & Y_{KE} \\ . & . & Y_{LL} & Y_{LE} \\ . & . & . & Y_{EE} \end{pmatrix} \begin{bmatrix} \partial\lambda \\ \partial K \\ \partial L \\ \partial E \end{bmatrix} = \begin{bmatrix} 0 \\ \partial r \\ \partial w \\ \partial e \end{bmatrix}. \tag{3}$$

This Hessian matrix is inverted to derive input cross price elasticities such as $\varepsilon_{Ke} = (\partial K/\partial e)(e/K)$ describing the local production frontier. The present approach evaluates these cross price elasticities at estimated marginal products.

The model assumes input prices are exogenous to the cost minimization. The US is arguably a price taker in global energy and capital markets. Wages are exogenous to the cost minimizing input decision due to labor contracts and market traditions.

The estimated physical production function (1) in natural log form is

$$\ln Y = \alpha_0 + \alpha_1(\ln L + \ln K) + \alpha_2(\ln E + \ln K) + \varepsilon \tag{4}$$

where ε is a white noise residual. Output elasticities are $\partial \ln Y/\partial \ln L \equiv \varepsilon_L = \alpha_1$ for labor, $\varepsilon_E = \alpha_2$ for energy, and $\varepsilon_K = \alpha_1 + \alpha_2$ for capital. With competitive input markets, output elasticities equal factor shares.

2. Data and pretests

Annual US data for 1951-2008 include real gross domestic product, real fixed capital assets, and the labor force, all from the NIPA National Income Product Accounts, and total Btu energy input from the Department of Energy.

Output Y in Figure 1 grows steadily with some irregularity and at a faster pace after the early 1980s. Capital K grows more regularly at a slightly increasing rate. Labor L generally grows but occasionally declines for brief periods. Energy E grows at a relatively fast pace up to the energy crisis when it declines before growing at a slower pace. The series are not stationary but prove difference stationary as suggested by Figure 2.

* Figure 1 * Figure 2 *

Table 1 reports Dickey-Fuller DF (1979) stationary tests. Output Y is difference stationary by the DF test with a constant and time trend. Corresponding tests for capital K and labor L have residual correlation according to Durbin-Watson DW (1951) statistics but are difference stationary in augmented Dickey-Fuller ADF tests. Energy E is more volatile with ARCH(1) residual heteroskedasticity but proves ADF difference stationary with six lags.

* Table 1 *

The difference stationary series suggest difference equation or perhaps error correction regressions. Regressions on levels produce suffer residual correlation and heteroskedasticity, while difference regressions are reliable. The following sections compare estimates of the log physical production function with log linear and translog.

3. Log linear production function estimates

The first column in Table 2 reports the log linear KL production function with capital and labor inputs. The labor effect appears exaggerated but standard errors are unreliable due to residual correlation. There is also residual heteroskedasticity according to the autoregressive conditional ARCH(1) test. The series are not cointegrated according to the Engle-Granger EG (1987) test, consistent with the insignificant error adjustment term of the error correction model KL ECM in the second column.

* Table 2 *

The KLt column reports a model assuming technology improves smoothly with year t added as an explanatory variable. This trend accounts for 2% output growth, disguising the importance of energy input as shown in the following estimates. The labor elasticity falls close to its factor share. Although series in the KLt model are not cointegrated according to the EG test, the associated error correction model KLt ECM reveals an error correction effect. Output elasticities derived as the transitory difference effect plus the error correction adjustment are 0.17 (0.10) for K and 1.07 (0.09) for L. The reported standard errors in parentheses are derived with error propagation calculations. The exaggerated labor effect is due omitted energy input. The sum of these coefficients 1.24 (0.13) suggests increasing returns that disappear when energy is included. Table 5 compares the derived own and cross elasticities for this KLt ECM with other models.

Adding energy input improves every corresponding KL model in Table 2 according to the Aikake (1973) Information Criteria AIC. In the KLE regression, residual correlation and heterskedasticity remain issues. The series are not cointegrated but the error correction model KLE ECM produces reliable statistics. Output elasticities for the KLE ECM are 0.12 (0.10) for capital, 0.69 (0.11) for labor, and 0.32 (0.08) for energy. Their sum 1.13 (0.17) is consistent with constant returns. The comparison in Table 5 includes this KLE ECM model.

The KLEt column in Table 2 adds smooth technology but residual correlation and heteroskedasticity remain problems. These series are not cointegrated but the associated KLEt ECM uncovers a strong error correction effect. Adding transitory and error correction effects, output elasticities are 0.76 (0.13) for labor and 0.41 (0.09) for energy while the capital output elasticity is insignificant. The sum of the coefficients 1.17 (0.17) is marginally consistent with constant returns. This model is superior to other regressions in Table 2 by the AIC but there remains 2% unexplained output growth in the constant term. Cross price elasticities cannot be estimated due to the zero (or negative) marginal product of capital.

4. Translog production function estimates

The translog production function of Christensen, Jorgensen, and Lau (1973) introduces interaction terms to the log linear specification opening the possibly of stronger substitution and complements in production. Table 3 reports estimates starting with the capital/labor translog KLT model. There appears to be capital-labor interaction but statistics are unreliable due to residual correlation and heteroskedasticity. The error correction model KLT ECM in the second column proves insignificant.

* Table 3 *

The third column in Table 3 adds smooth technology with the time trend in the KLTt model. Capital/labor interaction again appears possible but statistics remain unreliable. These series are more nearly cointegrated. Labor has strong first and second order effects in the associated error correction model KLTt ECM. There is also strong capital/labor interaction. This model has no economic meaning, however, with a negative capital output elasticity of -7.53 (3.32).

Adding energy input in Table 3 produces superior fit according to the AIC. The regression on levels in the KLET translog production function has apparent capital/labor interaction but residual correlation and heteroskedasticity remain. These series are cointegrated but the KLET ECM has no significant coefficients and is not considered in the model comparison.

Adding the technology trend produces the more successful KLETt model. The lack of residual heteroskedasticity implies energy input accounts for the typical unexplained output volatility. The error correction model KLETt ECM reduces the unexplained output growth to 1%

and is superior to all models by the AIC. Explanatory power is high but discounted somewhat by gray area Durbin-Watson residual correlation. There are strong capital/labor interaction and second order labor effects. The output elasticity for capital evaluated at sample means, however, is negative making the model unsuccessful.

Summarizing, only the error correction models KLt ECM and KLE ECM produce meaningful economic results. Translog estimates all violate economic assumptions. Unreported modified translog estimates with combinations of interaction and second order effects also fail to produce meaningful results.

5. Physical production function estimates

Table 4 reports estimates of the KL-KE physical production function (4). The regression on levels has residual correlation and heteroskedasticity. The series are not cointegrated by the Engle-Granger EG test but there is a significant error correction process in the associated error correction model.

* Table 4 *

The difference regression ΔKL-KE in the second column produces reliable estimates with no residual correlation or heteroskedasticity. The sum of the two coefficients in the difference equation 0.50 (0.17) is exactly constant returns to scale. Euler’s theorem follows with shadow factor shares of 0.12 (0.13) for labor, 0.38 (0.11) for energy, and 0.50 (0.17) for capital. Energy is underpaid and labor overpaid relative to their productivities.

The associated error correction model KL-KE ECM in the third column has a significant error correction process but gray area residual correlation. Derived output elasticities are 0.20 (0.12) for labor, 0.48 (0.12) for energy, and 0.68 (0.17) for capital, higher than those from the difference equation and marginally rejecting constant returns. As in the difference equation, energy is underpaid and labor overpaid.

The fourth column adds smooth technology t in the KL-KEt regression. The trend t accounts for 2% output growth. These series are not cointegrated by the Engle-Granger test but the associated KL-KEt ECM regression in the last column has a strong error correction process. There is no unexplained growth in the KL-KEt ECM model, superior to other physical production function estimates by the AIC. Derived output elasticities are 0.16 (0.11) for labor, 0.50 (0.10) for energy, and 0.66 (0.15) for capital, marginally rejecting constant returns. Energy is underpaid, and labor overpaid.

Figure 3 compares residuals of the KLEt and KL-KEt production functions on levels of variables with the two associated error correction models. Residuals have long periods of trending residuals, similar to log linear and translog models. Turning points are interpreted in the literature as changes in technology or total factor productivity. The residual of the physical production function KL-KEt has a different pattern and is closer to zero with a mean of -0.0002 (0.0033) compared to 0.0009 (0.0036) for KLEt. There is also slightly less residual correlation and heteroskedasticity with (ρ , ARCH F) equal to (0.72, 8.02) compared to (0.74, 8.82) for KLEt.

* Figure 3 *

Difference equation and error correction estimates eliminate trends and turning points in residuals. The mean of the KL-KEt ECM is closer to zero at -3.4E-18 (0.0019) compared to KLEt ECM at -64E-18 (0.0014). These two residuals are white noise while regressions on levels exhibit autocorrelation and heteroskedasticity.

Estimates of the physical production function with higher order interaction terms $(\ln L + \ln K)^2$, $(\ln E + \ln K)^2$, and $(\ln L + \ln K)(\ln E + \ln K)$ reveal no significant effects. Further, these second order effects have minimal impacts on estimates of α_1 and α_2. Inputs are substitutes due to this lack of interaction.

6. Comparing substitution elasticities

The present approach derives the own and cross price elasticities from marginal products in (2) and related second order terms in the Hessian matrix (3). The symmetric Hessian matrix of the error correction model KL-KEt ECM in Table 4 is

$$\begin{pmatrix} 0 & .7343 & .1199 & .4479 \\ . & -.0042 & .0014 & .0051 \\ . & . & -.0012 & .0014 \\ . & . & . & -.0031 \end{pmatrix}. \tag{5}$$

Invert (5) to derive partial derivatives of inputs with respect to input prices as cofactors of the inverse matrix. Relevant elements of the inverse matrix are

$$\begin{pmatrix} \partial K/\partial r & \partial L/\partial r & \partial E/\partial r \\ \partial K/\partial w & \partial L/\partial w & \partial E/\partial w \\ \partial K/\partial e & \partial L/\partial e & \partial E/\partial e \end{pmatrix} = \begin{pmatrix} -43.4 & 73.5 & 51.4 \\ 73.5 & 599 & 39.8 \\ 51.4 & 39.8 & -95.0 \end{pmatrix}. \tag{6}$$

Cross price elasticities are evaluated at sample means and marginal products in (5). The means and standard errors of the series entering elasticity calculations are 4.03 (0.07) for lnY, 3.92 (0.07) for lnK, 4.42 (0.04) for lnL, and 4.24 (0.04) for lnE. For instance, the cross price elasticity of energy input with respect to the wage is

$$\varepsilon_{Ew} = (\partial E/\partial w)w/E = (\partial E/\partial w)Y_L/E_\mu. \tag{7}$$

Table 5 compares derived cross price elasticities in four models. The four entries are KLt ECM, KLE ECM, ΔKL-KE, and KL-KEt ECM elasticities as in (6).

* Table 5 *

Comparison of the two log linear models KLt ECM and KLE ECM shows omitting energy understates own labor substitution. A good deal of labor policy analysis focuses on own labor substitution elasticities that are biased downward in estimates excluding energy input. Capital substitution relative to the wage is overstated when energy input is omitted. Substitution of energy accounts for some of the apparent capital substitution in models without energy.

In the physical production functions ΔKL-KE and KL-KEt ECM capital is a weaker own substitute than in the log linear KLE ECM. Capital interaction with the two "force" inputs makes capital more critical in production and less sensitive to its own price. Labor is a slightly stronger own substitute in the physical production functions relative to the log linear KLE ECM.

Regarding changes in the capital price, substitution for labor and energy is much stronger in the physical production function. Separate interaction of the two inputs increases their sensitivity to the capital price. Relative to the wage, there is weaker capital substitution given its link to energy input. There is also much weaker substitution of energy relative to the wage.

Table 5 also reports the change in marginal cost as the Lagragian multilplier λ in (3) relative to each factor price in the ∂MC column. The ε_{MC} column reports corresponding elasticities of marginal cost evaluated at sample means with MC equal to one and output as national product. Comparing the two log linear models, excluding energy overstates the effects of the wage and capital price on marginal cost. A portion of the apparent effects in the KLt ECM model is due to energy adjustment as shown by KLE ECM, especially for capital.

The physical production functions in the last two entries imply greatly reduced wage elasticities of marginal cost. Allowing capital to interact with energy separately from labor reduces the effects of wages on marginal cost.

The last column of Table 5 reports the derived marginal products MP. Comparing log linear models in the first two entries, omitting energy overstates marginal products of capital and labor. Relative to log linear models, the physical production functions have much lower marginal products of labor and higher marginal products of capital.

Separate capital interaction with the two inputs increases the marginal productivity of capital. The marginal product of energy is similar across models, consistently three to four times its factor share.

7. Conclusion

Log linear production functions are theoretically sensible and successful as first order approximations. Translog production functions improve empirical fit introducing second order interaction effects. The present physical production function provides an alternative functional form that produces superior estimates in the present aggregate US time series. The present results strongly suggest including energy input as a primary factor for aggregate output. The stakes are high as the neoclassical production function lays the foundation for economic growth and macroeconomics.

Energy input affects estimated substitution between capital and labor. Energy has a higher marginal product than labor as well as the largest effect on marginal cost. In stark contrast, the economy readily absorbs changes in the capital price suggesting policy reactions to the recent financial crisis were not economic.

Energy input raises a host of policy issues. The evidence that energy is underpaid suggests the US has monopsony power in the global energy market. An oil tariff would then lower the world price, perhaps lowering the domestic price including the tariff in the Metzler (1949) effect. A variable tariff rate targeting a stationary domestic energy price would weaken disruptive variability in energy prices. The high shadow price of energy implies the regulatory constraints on franchise monopoly utilities are binding. As a final implication, there is underinvestment in energy alternatives based on market prices rather than shadow prices.

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Figure 1. Trends in Output and Inputs

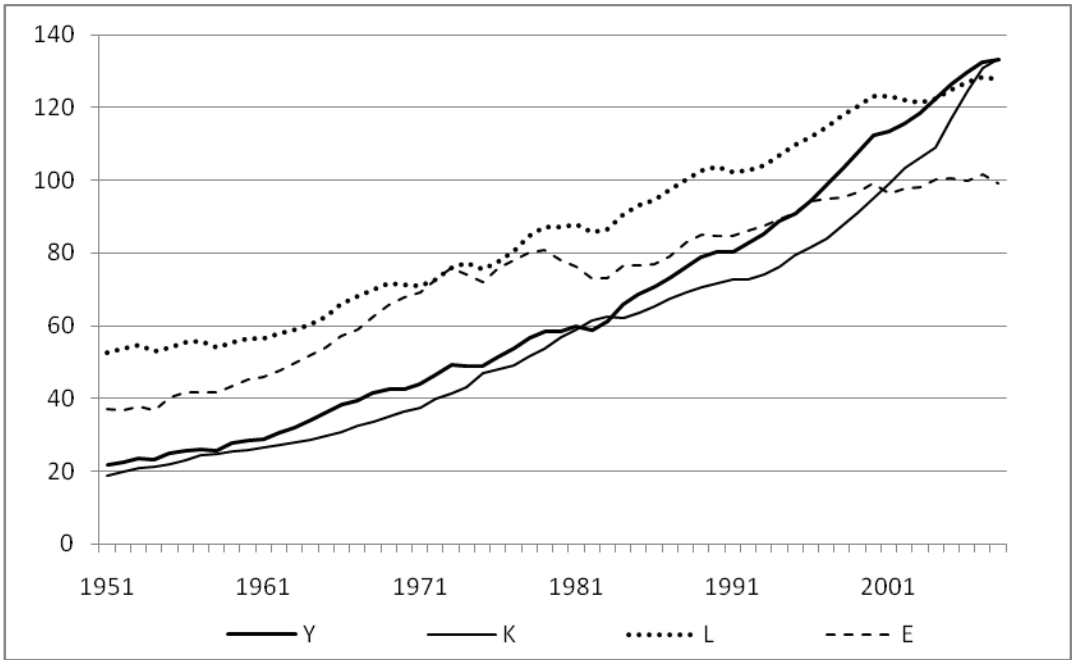

Figure 2. Differences in Inputs and Output

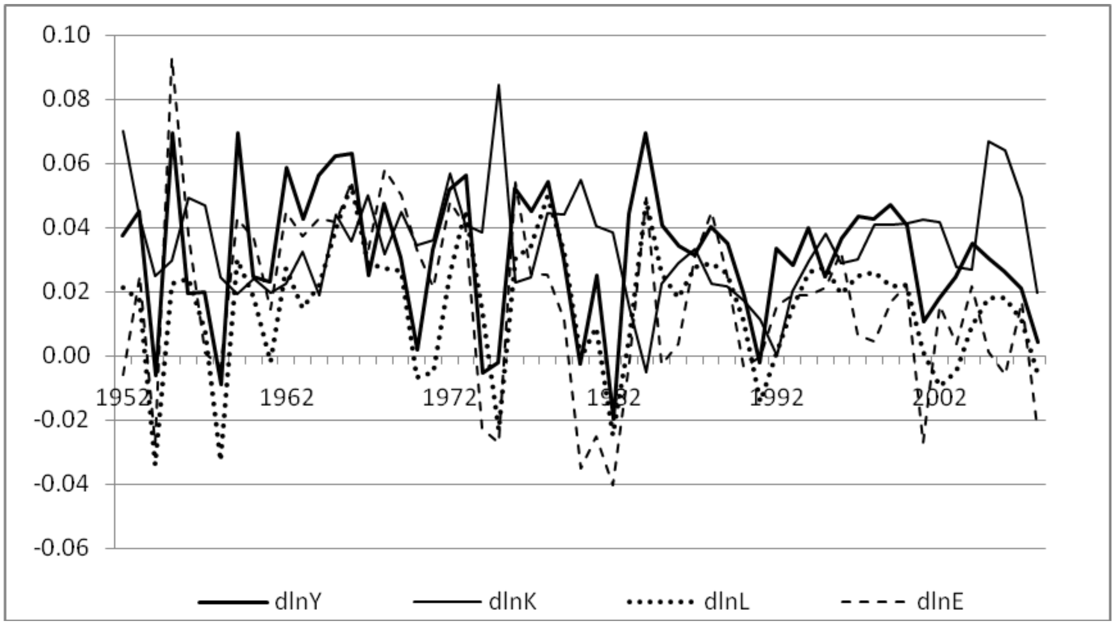

Figure 3. Production Function Residuals

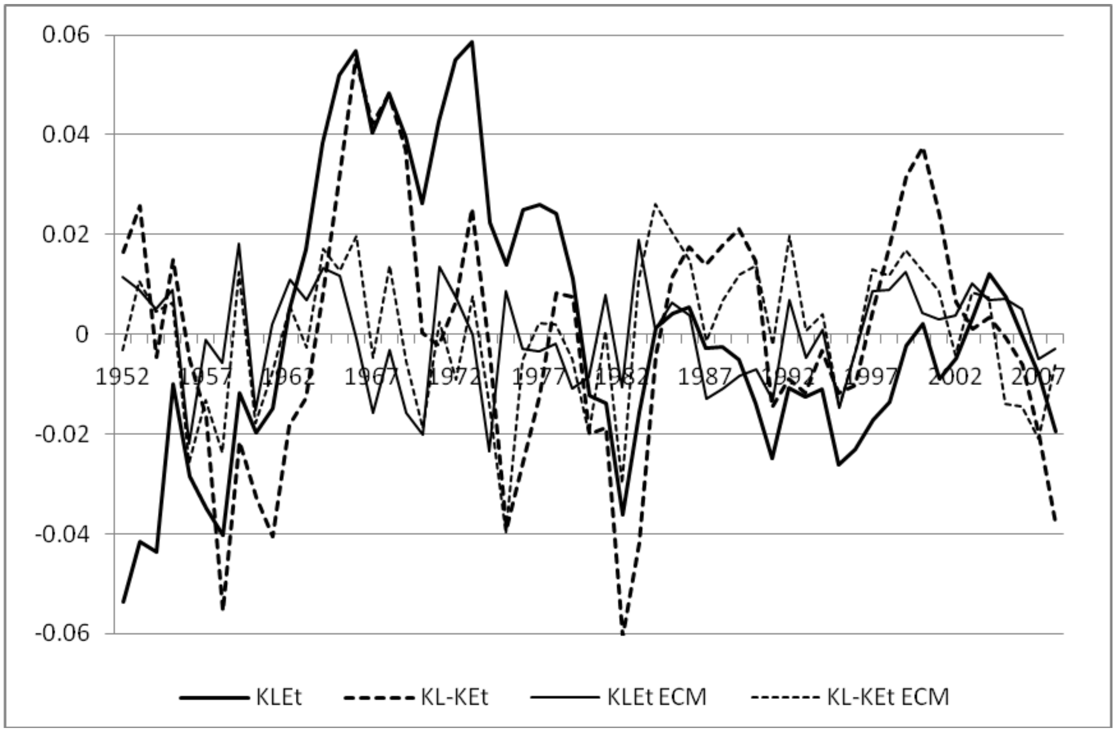

Table 1. Stationarity Analysis

τ_{DF} -3.80	DF	ADF	ADF(6)
lnY	-2.83		
ϕ 6.73	6.01		
DW 1.65	1.68		
ARCH F	0.18		
lnK	-1.38	-1.72	
ϕ	1.01	4.40	
DW	1.18*	1.96	
ARCH F	0.91	0.48	
lnL	-2.16	-3.10	
ϕ	3.09	5.03	
DW	1.38*	1.98	
ARCH F	-1.61	-0.57	
lnE	-1.32	-1.01	-1.13
ϕ	6.05	3.35	2.20
DW	1.68	2.00	1.80
ARCH F	4.06*	2.64*	-0.20

Table 2. Log Linear Production Functions

lnY	KL	KL ECM	KLt	KLt ECM	KLE	KLE ECM	KLEt	KLEt ECM
constant	-2.38***	0.03***	-35.8***	0.02***	-2.44***	0.02***	-45.7***	0.02***
(SE)	(.305)	(0.004)	(0.05)	(0.004)	(0.30)	(0.004)	(3.33)	(0.003)
K	0.40***	-0.23**	0.05	0.16	0.37***	-0.10	-0.09	-0.04
	(0.07)	(0.10)	(0.08)	(0.10)	(0.06)	(0.10)	(0.05)	(0.08)
L	1.09***	0.89***	0.71***	0.96***	0.96***	0.64***	0.39***	0.64***
	(0.13)	(0.09)	(0.12)	(0.08)	(0.12)	(0.10)	(0.07)	(0.09)
E					0.17***	0.31***	0.27***	0.32***
					(0.05)	(0.08)	(0.03)	(0.06)
t			0.02***				0.02***	
			(0.003)				(0.002)	
EC res		-0.01		-0.16***		-0.05		-0.32***
		(0.05)		(0.06)		(0.05)		(0.08)
R^2	.996	.689	.997	.727	.996	.763	1.00	.814
DW	0.20*	1.87	0.23*	1.71	0.20*	1.84	0.51*	1.57^g
ARCH F	74.9*	0.04	54.1*	1.58	97.1*	0.22	21.5*	0.09
AIC	-143	-271	-173	-278	-153	-284	-232	-298
EG	-2.05		-2.32		-1.76		-2.97	

Table 3. Translog Production Functions

lnY	KLT	KLT ECM	KLTt	KLTt ECM	KLET	KLET ECM	KLETt	KLETt ECM
Constant	-33.1* (19.5)	0.03*** (0.004)	-88.9*** (12.5)	0.02*** (0.004)	-27.6*** (10.4)	0.02*** (0.004)	-62.5*** (7.34)	0.01*** (0.004)
K	-14.4* (8.62)	-3.18 (3.32)	-10.4** (5.01)	-4.93 (3.14)	-14.1*** (4.60)	-4.18 (3.13)	-11.3*** (2.79)	-7.30*** (2.73)
L	28.2* (16.4)	8.22 (6.32)	24.1** (9.53)	12.5** (6.06)	18.6** (8.57)	6.33 (5.96)	18.1*** (5.18)	13.1** (5.28)
E					7.00** (2.86)	2.21 (1.77)	4.76*** (1.74)	1.56 (1.50)
K^2	-1.74* (0.95)	-0.33 (0.36)	-1.02* (0.55)	-0.46 (0.33)	-1.50*** (0.51)	-0.42 (0.34)	-1.13*** (0.31)	-0.70** (0.30)
L^2	-5.93* (3.47)	-1.40 (1.33)	-4.50** (2.01)	-2.18* (1.26)	-4.34* (1.92)	-0.92 (-1.44)	-3.36*** (1.17)	-2.93** (1.30)
E^2					0.79* (0.45)	0.22 (0.35)	0.06 (0.29)	-0.08 (0.31)
KL	6.45* (3.63)	1.26 (1.39)	4.13* (2.11)	1.92 (1.31)	5.61*** (1.88)	1.62 (1.33)	4.49*** (1.14)	3.27*** (1.19)
KL					0.32 (0.55)	0.05 (0.39)	0.07 (0.33)	-0.38 (0.34)
LE					-0.39 (1.30)	-0.90 (0.97)	-1.22 (0.79)	0.19 (0.22)
t			0.03*** (0.003)				0.02*** (0.002)	
EC res		-0.01 (0.05)		-0.25*** (0.09)		-0.10 (0.12)		-0.72*** (0.17)
R^2	.996	.708	.999	7.46	1.00	.780	1.00	.843
DW	0.22*	1.98	0.44*	1.75	0.91*	1.94	1.25*	1.56^g
ARCH F	57.6*	0.05	17.7*	0.10	5.15*	-0.12	0.001	0.41
AIC	-145	-268	-207	-277	-219	-284	-278	-303
EG	-1.75		-2.77		-4.11*		-5.13*	

Table 4. Physical Production Functions

lnY	KL-KE	ΔKL-KE	KL-KE ECM	KL-KEt	KL-KEt ECM
constant	-1.28*** (0.05)	0.006 (0.005)	0.001 (0.006)	-44.7*** (5.07)	0.001 (0.005)
K+L	0.49*** (0.06)	0.12 (0.13)	0.14 (0.12)	-0.08 (0.07)	0.19* (0.11)
K+E	0.15** (0.06)	0.38*** (0.11)	0.46*** (0.12)	0.25*** (0.04)	0.41*** (0.10)
t				0.02*** 0.003	
EC res			-0.13*** (0.07)		-0.37*** (0.08)
R^2	.996	.381	.425	.998	.522
DW	0.22*	1.72	1.59*	1.59*	1.45*
ARCH F	7.78*	2.78	-0.03	7.19*	0.05
EG	-1.74			-2.83	
AIC	-139	-234	-236	-185	-250

Table 5. Cross Price Substitution Elasticities

KLt ECM KLE ECM ΔKL-KE KL-KEt ECM	∂lnK	∂lnL	∂lnE	∂MC	ε_{MC}	MP	
∂lnr	-0.86	0.14		0.72	0.14	Y_K	0.19
	-0.98	0.03	0.01	1.18	0.02		0.02
	-0.53	0.59	0.51	0.85	0.07		0.55
	-0.54	0.62	0.52	0.62	0.05		0.73
∂lnw	0.86	-0.14		1.08	0.86	Y_L	0.80
	0.21	-0.61	1.16	1.50	0.77		0.51
	0.14	-0.84	0.08	1.58	0.14		0.09
	0.15	-0.83	0.07	1.22	0.15		0.12
∂lne						Y_E	
	0.39	0.25	-0.59	0.66	0.20		0.31
	0.21	0.59	-1.17	1.15	0.39		0.34
	0.39	0.21	-0.59	0.58	0.20		0.45