Some Suggestions for Reading GTM 52

I. What is GTM 52?

GTM 52 refers to *Algebraic Geometry*, a textbook published in 1977 by the algebraic geometer Robin Hartshorne, as part of the *Graduate Texts in Mathematics* series. The book is organized into five chapters:

1. Varieties
2. Schemes
3. Cohomology
4. Curves
5. Surfaces

The core content introduces essential concepts and theorems in algebraic geometry using Grothendieck's scheme language, along with classical results concerning algebraic curves and surfaces via cohomological methods.

II. Why Read GTM 52?

GTM 52 is often referred to as the *"Bible"* of algebraic geometry. Its definitions and conventions have become the standard in modern algebraic geometry. Despite being under 500 pages, the book is exceptionally dense and precise, with minimal errors, making it a classic among mathematical textbooks.

Students studying algebraic geometry and related fields should consider using this book as a primary resource. However, it is important to note that reading the entire book may take about a year for students with some background knowledge. If your goal is only to gain a basic understanding of algebraic geometry, there are other more accessible books, which I will recommend later.

III. Prerequisites

Before diving into GTM 52, you should have a solid understanding of:

- **Differential geometry**

- **Algebraic topology**
- **Commutative algebra** (e.g., topics like complex manifolds, vector bundles, (de Rham) cohomology, and normalization).

Particularly, a good grasp of commutative algebra is essential, and it is recommended to study from *Atiyah-Macdonald's Introduction to Commutative Algebra*. Additionally, you should have some preliminary knowledge of algebraic geometry. GTM 52 is not suitable as a beginner's first book on the subject. I will suggest some introductory books later.

IV. How to Approach the Book

Due to its reliance on Grothendieck's abstract scheme and category language, and the author's minimal use of explanatory text regarding the geometric intuition behind the concepts, GTM 52 can be quite challenging. One key piece of advice I received from an experienced professor is: **don't read the book linearly from start to finish**. The professor said, *"Reading linearly is only for undergraduates."*

In practice, this means you shouldn't insist on fully understanding every section before moving on. If you get stuck, read through the section carefully, but feel free to skip ahead and come back later. Trying to read the book cover to cover might take a decade, and you could end up unclear about what you truly understand. When you encounter difficulties, just think it through to the best of your ability, and revisit the material later if needed.

As for the content:

- **Chapter 1** serves as a warm-up, providing many concrete examples that will aid your understanding of later chapters. You should refer back to these examples often.
- **Chapters 2 and 3** are the most difficult and will likely require multiple readings. I recommend initially reading through Chapter 2 up to *Differentials* and Chapter 3 up to *Serre duality*. After that, you can move on to Chapter 4.
- Chapters 2 and 3 introduce abstract theories that are applied in Chapters 4 and 5. Seeing the applications will help you grasp the earlier abstract material more concretely.
- You can skip the section on **Formal Schemes** in Chapter 2, as it is rarely applied later.
- The sections after *Serre duality* in Chapter 3 are particularly challenging and not crucial to understanding the rest of the book, so it is advisable to skip them on the first reading.

Each section concludes with exercises. It's beneficial to complete as many exercises as possible—ideally:

- **80% of Chapter 1**
- **35% of Chapters 2 and 3**
- **20% of Chapters 4 and 5**

Many of the theorems in the book reference results from the exercises, so I recommend completing any exercises that are cited in the proofs. If you get stuck on an exercise, move on and return to it later.

V. Recommended Reference Books

1. **EGA (*Éléments de Géométrie Algébrique*)**:
   Grothendieck's *EGA* is undoubtedly a timeless classic. While GTM 52 covers much of the material in EGA, Grothendieck's groundbreaking insights can only be fully appreciated through the original text. For example, a scheme X can be viewed as a functor from affine schemes (or rings) to sets: $\mathrm{Hom}(\mathrm{Spec}(R), X)$. A point on a scheme corresponds to an element of $\mathrm{Hom}(\mathrm{Spec}(R), X)$, where R is any commutative ring. This understanding lays the foundation for deformation theory and moduli spaces. Such abstract category-theoretic language not only simplifies proofs but also provides a higher-level perspective on problems. However, EGA is even more challenging to read, so it is best tackled after GTM 52.

2. **Basic Algebraic Geometry, Volumes 1 & 2 by Shafarevich**:
   These textbooks, written around the same time, are excellent resources. Volume 1 is essentially an expanded version of GTM 52's first chapter, while Volume 2 covers schemes and complex manifolds. Though the coverage is broad rather than deep, the detailed exposition makes these books highly recommended. They are particularly suitable for students from other fields who wish to gain a basic understanding of algebraic geometry.

3. **Principles of Algebraic Geometry by P. Griffiths and J. Harris**:
   This beautiful book uses complex geometry to study algebraic geometry and is the only comprehensive textbook on the topic. Before A. Weil and O. Zariski, mathematicians used complex geometric language to study algebraic geometry, and this book offers insights into their early ideas. These ideas are obscured by the abstract language in GTM 52. However, the book contains many small errors and is not suitable for beginners. Interested readers might first explore R. O. Wells' *Differential Analysis on Complex Manifolds*.

4. **Other Recommended Textbooks**:
   Although I haven't explored these in detail, they are well-regarded:

   - David Mumford: *Complex Projective Varieties*; *The Red Book of Varieties and Schemes*

- Joe Harris: *Algebraic Geometry: A First Course*
- Joe Harris & David Eisenbud: *The Geometry of Schemes*
- James Milne: *Algebraic Geometry*, with lecture notes available at jmilne.org. I've found his notes very helpful.

Finally, if you are interested in algebraic geometry but lack some of the prerequisite knowledge (especially in commutative algebra), consider starting with a book on **Riemann surfaces**. It provides a sense of what algebraic geometry is about and serves as a gentler introduction.