

**Final Project**

For one given irreducible root system  $\Phi$ , do the following tasks:

1. Explicitly describe a matrix Lie algebra  $L$  that associates with the root system  $\Phi$ .
2. Determine a maximal toral subalgebra  $H$  of  $L$ . Then describe the root space decomposition of  $L$  relative to  $H$ . Explicitly describe the roots in  $\Phi$  and the corresponding root spaces.
3. Choose a base  $\Delta$  of  $\Phi$ , and describe the corresponding  $\Phi^+$ .
4. Express roots in  $\Phi^+$  as integral combinations of simple roots in  $\Delta$ . What is the maximal root?
5. What is the Dynkin diagram and the Cartan matrix relative to  $\Delta$ ? Label the elements of  $\Delta$  in the Dynkin diagram.
6. Describe the Weyl group  $\mathcal{W}$  up to isomorphism.
7. Determine the fundamental Weyl chamber of  $\Phi$  associate to  $\Delta$ .
8. How many Weyl chambers in  $\Phi$ ? How many bases?