Periodic solutions and connecting orbits for nonlinear evolution equations at resonance

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Abstract

Assume that $A : X \supset D(A) \to X$ is a positive definite sectorial operator on a Banach space X and let X^α, where $\alpha \in [0, 1)$, be a fractional power space given by $X^\alpha := D(A^\alpha)$. We shall consider differential equations of the form

$$\dot{u}(t) = -Au(t) + \lambda u(t) + F(t, u(t)), \qquad t > 0 \tag{1}$$

$$\ddot{u}(t) = -Au(t) - cA\dot{u}(t) + \lambda u(t) + F(t, u(t)), \qquad t > 0 \tag{2}$$

where $c > 0$, λ is a real number and $F : [0, +\infty) \times X^\alpha \to X$ is a continuous map. Our goal is study the existence of T-periodic solutions ($T > 0$) and orbits connecting stationary points for the above equations being at *the resonance at infinity*, that is, $\ker(\lambda I - A) \neq \{0\}$ and F is bounded. The main difficulty lies in the fact that, due to the presence of resonance, the equations (**??**) and (**??**) may not have periodic solutions and bounded orbits for general perturbation F. Therefore we formulate geometrical conditions characterizing nonlinearity F and use them to prove theorems determining the existence of T-periodic solutions and orbits connecting stationary points for equations (**??**) and (**??**). The methods that we will use involve the application of homotopy invariants such as topological degree and Conley index to the semiflows associated with these equations. Finally, we provide applications of the obtained results in the case when A is second order differential operator on $X := L^p(\Omega)$, where $\Omega \subset \mathbb{R}^n$ is a bounded set and F is a Niemycki operator associated with continuous map $f : [0, +\infty) \times \Omega \times \mathbb{R} \to \mathbb{R}$. In particular, we prove that introduced geometrical assumptions generalize well known *Landesman–Lazer* and *strong resonance* type conditions.

References

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