

Annotated Examples 9

Reducible

From my point of view, one should decouple reducibility from integral closure, in that each separate affine domain problem can then be treated independently, meaning from a possibly very different perspective in terms of weights, etc.

That said, let's consider a simple reducible example defined by

$$(y^3 - yx - x^5)(y^2 - x^9 - x^2) = 0$$

(in nearly any characteristic except 2 or 7). The first factor would produce an integral closure with $wt(x) = 3$, $wt(y) = 5$ and $wt(y^2/x) = 7$; the second one with $wt(x) = 2$, $wt(y/x) = 7$.

If we don't notice that it is *reducible*, then we naively get an answer with $P := \mathbf{F}[x]$ -module basis

$$\begin{aligned} y_0 &:= 1, \quad y_1 := y, \quad y_2 := y^2/x, \quad y_3 := (y^3 - yx)/x^2, \\ y_4 &:= (y^3 - yx - x^5) * (y + x^{24} + x^{23} + x^{22} + 3x^{17} + x^{16} + x^{15} - x^{14} + 3x^{10} - x^9 - 2x^7 - x^6 - x^5 + x^3 - x^2) / D \\ D &:= x^4(x^{23} + 3x^{16} - 2x^{15} + 3x^9 - 4x^8 + x^7 - x^6 + x^2 - 2x + 1) \end{aligned}$$

This reduces $mod(y^3 - yx - x^5)$ to $(1, y, y^2/x)$ and $mod(y^2 - x^9 - x^2)$ to $(1, y/x)$, as mentioned above.

Conversely, it is possible to reconstruct these 5 basis elements from the two smaller bases, by using the Chinese Remainder Theorem on the two moduli.

$$\begin{aligned} &-(y^2 - x^9 - x^2)(y^2(x^9 + x^2 - x) + yx^5 + (x^9 + x^2 - x)^2) + (y^3 - yx - x^5)(y(x^9 + x^2 - x) + x^5) \\ &= (x^9 + x^2 - x)^2(x^9 + x^2) - x^10 = D. \end{aligned}$$

But at least from this, we know theoretically, in simple terms, why the integral closure can be viewed as the product of separate integral closures and how one can move back and forth between the two types of answers.

Now consider what SINGULAR produces:

```
SINGULAR /
A Computer Algebra System for Polynomial Computations / version 3-1-0
0<
by: G.-M. Greuel, G. Pfister, H. Schoenemann \ Mar 2009
FB Mathematik der Universitaet, D-67653 Kaiserslautern \
> LIB ".../presolve.lib";
> LIB ".../normal.lib";
> ring r=0,(y,x),lp;
> ideal i=(y^3-y*x-x^5)*(y^2-x^9-x^2);
> list nor=normal(i);
// Ri/norid is the affine algebra of the normalization of the i-th
```

```

// component r/P_i (where P_i is the i-th component of a decomposition of
// the input ideal) and normap the normalization map from r to Ri/norid.
> def R1=nor[1][1];
> setring R1;
> normap;
normap[1]=y
normap[2]=x
> norid;
norid[1]=-T(1)*x+y
norid[2]=-T(1)*y+x^8+x
norid[3]=T(1)^2-x^7-1
norid[4]=-y^2+x^9+x^2
> setring r;
> def R2=nor[1][2];
> setring R2;
> normap;
normap[1]=y
normap[2]=x
> norid;
norid[1]=-T(1)*x+y^2
norid[2]=-T(1)*y+y+x^4
norid[3]=T(1)^2-T(1)-y*x^3
norid[4]=-y^3+y*x+x^5

```

We must know how to read P -module generating sets from these presentations, if we wish to reconstruct the naive integral closure. Perhaps we don't care about this reconstruction, and are happy with these two answers instead; but we should ask in what sense they form the integral closure/normalization—my answer being as above, that these two separate problems should have been separated far in advance of computing any integral closure.

MAGMA's IntegralClosure won't work because it requires a FunctionField; but its Normalisation will:

```

> Q:=Rationals();
> P<y,x>:=PolynomialRing(Q,2);
> f:=(y^3-y*x-x^5)*(y^2-x^9-x^2);
> I:=ideal<P|f>;
> J:=Normalisation(I);
> K1:=J[1][1];K1;
Ideal of Polynomial ring of rank 3 over Rational Field
Order: Lexicographical
Variables: $.1, $.2, $.3
Basis:
[
  -$.1*$.3 + $.2^2,
  -$.1^4 + $.2*$.3 - $.2,
  -$.1^3*$.2 + $.3^2 - $.3

```

```

]
> K2:=J[2][1];K2;
Ideal of Polynomial ring of rank 2 over Rational Field
Order: Lexicographical
Variables: $.1, $.2
Basis:
[
  -$.1^7 + $.2^2 - 1
]

```

It clearly gives P -module bases for both individual subrings; but, as noted, before, its monomial ordering of choice is terrible.

It used to be the case that MACAULAY 2 claimed it didn't work on such non-affine problems. That still seems to be true, though the documentation claims otherwise. [I assume (thus making an "ass" out of "u" and "me") that this happened because there was new code written, documented, and shelved without undocumenting. Clearly I could be wrong about this.]

```

Macaulay 2, version 1.2
with packages: Elimination, IntegralClosure, LLLBases, PrimaryDecomposition,
               ReesAlgebra, SchurRings, TangentCone

```

```

i1 : load "/home/leonada/IntegralClosure.m2";

i2 : S=QQ[y,x,MonomialOrder=>{Lex}];

i3 : I=ideal((y^3-y*x-x^5)*(y^2-x^9-x^2));

o3 : Ideal of S

i4 : R=S/I;

i5 : time P=presentation(integralClosure(R))
      -- used 0.56 seconds

o5 = | y5-y3x9-y3x2-y3x-y2x5+yx10+yx3+x14+x7 |

      1      1
o5 : Matrix S  <--- S

i6 : S=ZZ/23[y,x,MonomialOrder=>{Lex}];

i7 : I=ideal((y^3-y*x-x^5)*(y^2-x^9-x^2));

o7 : Ideal of S

```

i8 : R=S/I;

i9 : time icF=icFracP(R); -- used 24081.2 seconds

i10 : toString(o9);

$$\frac{(-9y^4+2y^3x^4+9y^2x+7y^2x^5-2x^9)}{(y^4+10y^3x^4-y^2x-8y^2-11y^2x^5-2x^9+8x^2)},$$

$$\frac{(-9y^4+2y^3x^4+9y^2x-2y^2+7y^2x^5+2x^2)}{(y^4+10y^3x^4-y^2x-8y^2-11y^2x^5-2x^9+8x^2)},$$

$$\begin{aligned} & (10y^4x^{22}+8y^4x^{21}+4y^4x^{20}-y^4x^{19}+11y^4x^{18}-6y^4x^{17} \\ & -y^4x^{16}+4y^4x^{15}+7y^4x^{14}-8y^4x^{13}+10y^4x^{12}+9y^4x^{11} \\ & -3y^4x^{10}+8y^4x^9+8y^4x^8+4y^4x^7-5y^4x^6-y^4x^5+10y^4x^4 \\ & +9y^4x^3-4y^4x^2+y^4x-9y^4+5y^3x^{26}+7y^3x^{25}+11y^3x^{24} \\ & -3y^3x^{23}+10y^3x^{22}-y^3x^{21}-9y^3x^{20}+7y^3x^{19}+11y^3x^{18} \\ & +10y^3x^{17}+10y^3x^{16}-8y^3x^{15}-6y^3x^{14}+5y^3x^{12}+6y^3x^{11} \\ & +6y^3x^{10}+3y^3x^9-11y^3x^8+3y^3x^7+6y^3x^6-10y^3x^4 \\ & -6y^3x^2+y^3x+7y^3+8y^2x^{30}+8y^2x^{29}+3y^2x^{27}-8y^2x^{26} \\ & +9y^2x^{25}-5y^2x^{24}-8y^2x^{23}-3y^2x^{22}-5y^2x^{21}-2y^2x^{20} \\ & -4y^2x^{19}+y^2x^{18}-11y^2x^{17}+y^2x^{16}+4y^2x^{15}+6y^2x^{14} \\ & +11y^2x^{13}-2y^2x^{12}-9y^2x^{11}-4y^2x^{10}-10y^2x^9+3y^2x^8 \\ & -11y^2x^7+11y^2x^6-4y^2x^5+11y^2x^4-9y^2x^3-3y^2x^2 \\ & -8y^2x+4y^2+11y^2x^{31}-6y^2x^{30}+8y^2x^{29}-6y^2x^{28}-7y^2x^{27}-7y^2x^{26} \\ & -4y^2x^{25}+11y^2x^{24}-7y^2x^{23}+9y^2x^{22}-8y^2x^{21}+7y^2x^{20}-8y^2x^{19} \\ & -4y^2x^{18}+9y^2x^{17}-7y^2x^{16}-10y^2x^{15}-8y^2x^{14}-6y^2x^{13}+5y^2x^{12} \\ & -8y^2x^{11}+5y^2x^{10}+5y^2x^9+8y^2x^8+7y^2x^7+11y^2x^6-2y^2x^5-11y^2x^4 \\ & -y^2x^3+7y^2x^2+11y^2x-7x^{32}+6x^{31}-7x^{30}-2x^{29}+3x^{28}-3x^{26} \\ & -11x^{25}-8x^{24}-11x^{23}-3x^{22}+7x^{21}+7x^{20}-7x^{19}-10x^{18}-5x^{17} \\ & +x^{16}+10x^{15}+5x^{14}-3x^{13}+5x^{12}-6x^{11}-x^{10}-6x^9+4x^8+8x^7+4x^6 \\ & -7x^5+9x^4-4x^3-9x^2) \\ & / (y^4x^{18}+y^4x^{17}-8y^4x^{16}+5y^4x^{15}+3y^4x^{14}-10y^4x^{13} \\ & -7y^4x^{12}+7y^4x^{11}+8y^4x^{10}-y^4x^9+6y^4x^8+8y^4x^7 \\ & -10y^4x^6-9y^4x^5-3y^4x^4-3y^4x^3+7y^4x^2-5y^4x+2y^4 \\ & -y^3x^{23}-10y^3x^{22}+6y^3x^{21}+3y^3x^{20}-11y^3x^{19}-y^3x^{18} \\ & -2y^3x^{17}-5y^3x^{15}-10y^3x^{14}+7y^3x^{13}-6y^3x^{12}-2y^3x^{11} \\ & -10y^3x^{10}-8y^3x^9-9y^3x^8+7y^3x^7+9y^3x^6-4y^3x^5 \\ & +10y^3x^4+2y^3x^2+9y^3x+3y^3-3y^2x^{20}+8y^2x^{19}-4y^2x^{18} \\ & -11y^2x^{17}+9y^2x^{16}-y^2x^{15}-7y^2x^{14}+11y^2x^{13}+8y^2x^{12} \\ & -2y^2x^{11}-7y^2x^{10}-10y^2x^9+10y^2x^8+9y^2x^7+10y^2x^6 \\ & +2y^2x^5-5y^2x^4+10y^2x^3-7y^2x^2+4y^2x-3y^2+y^2x^{24}-4y^2x^{23} \\ & -8y^2x^{22}-6y^2x^{21}+4y^2x^{20}-10y^2x^{19}-6y^2x^{18}+11y^2x^{17}+6y^2x^{16} \\ & -4y^2x^{15}-9y^2x^{14}-2y^2x^{13}+8y^2x^{12}-3y^2x^{11}-6y^2x^{10}+8y^2x^9-7y^2x^8 \end{aligned}$$

```

+10*y*x^7+9*y*x^6+7*y*x^5+7*y*x^4-6*y*x^3+11*y*x^2+7*y*x+x^28+10*x^27
-6*x^26-3*x^25-4*x^24+10*x^23+3*x^22-4*x^21-3*x^20+4*x^19+7*x^18+5*x^17
-4*x^16+11*x^15-9*x^14-7*x^13+10*x^12-11*x^11+4*x^9+4*x^8-7*x^7-6*x^6
-8*x^5-x^4-3*x^3+x^2),

```

```

(-9*y^4-8*y^3*x^7+5*y^3*x^6-6*y^3*x^5-4*y^3*x^4-9*y^3*x^3-3*y^3*x^2
-y^3*x+9*y^2*x+8*y*x^8-5*y*x^7+6*y*x^6-10*y*x^5+9*y*x^4+3*y*x^3
+y*x^2+8*x^12-5*x^11+6*x^10+4*x^9+9*x^8+3*x^7+x^6)
/(y^3*x-y*x^2-x^6)

```

Note that integralClosure found nothing; and in the *icFracP* answer, at least the last fraction has a zero-divisor $(y^3 - yx - x^5)$ in the denominator.

My characteristic 0 program in MAGMA can be tricked into doing this by choosing the weight of y to be sufficiently larger than the weight of x , to make this work almost like a *lex ordering* problem instead of a weighted one. Absent a factorization step, one would never notice that this was reducible, and maybe also that the “weights” weren’t really weights.

```

leonada@cactus$ magma intclos_input081709.mag
Magma V2.15-9 Thu Aug 27 2009 13:15:08 on cactus [Seed = 1756688154]
Loading file "intclos_input081709.mag"
Loading "intclos_func70109.mag"
Loading "intclos_func070109.mag"

```

```

f=$.1^5-$$.1^3*$.2^9-$$.1^3*$.2^2-$$.1^3*$.2-$$.1^2*$.2^5+$$.1*$.2^10
+$.1*$.2^3+$.2^14+$.2^7

```

```

-----
time for q= 3 is 2.080 seconds
modulus= 3

```

```

-----
time for q= 5 is 1.580 seconds
modulus= 15

```

```

-----
time for q= 7 is 28.400 seconds
case4

```

```

-----
time for q= 11 is 1.160 seconds
modulus= 165

```

```

1 $.1^4+$$.1^3*$.2^23+$$.1^3*$.2^22+$$.1^3*$.2^21+3*$.1^3*$.2^16
+$.1^3*$.2^15+$$.1^3*$.2^14-$$.1^3*$.2^13+3*$.1^3*$.2^9-$$.1^3*$.2^8
-2*$.1^3*$.2^6-$$.1^3*$.2^5-$$.1^3*$.2^4+$$.1^3*$.2^2-$$.1^3*$.2
-$$.1^2*$.2-$$.1*$.2^24-$$.1*$.2^23-$$.1*$.2^22-3*$.1*$.2^17-$$.1*$.2^16
-$$.1*$.2^15+$$.1*$.2^14-3*$.1*$.2^10+$$.1*$.2^9+2*$.1*$.2^7

```

```

+$$.1*.2^6-$.1*.2^3+$.1*.2^2-$.2^18-2*$.2^11+$.2^10-$.2^4+$.2^3
2 $.1^3*$.2^24+3*$.1^3*$.2^17-2*$.1^3*$.2^16+3*$.1^3*$.2^10
-4*$.1^3*$.2^9+$.1^3*$.2^8-$.1^3*$.2^7+$.1^3*$.2^3-2*$.1^3*$.2^2
+$.1^3*$.2-$.1*.2^25-3*$.1*.2^18+2*$.1*.2^17-3*$.1*.2^11
+4*$.1*.2^10-$.1*.2^9+$.1*.2^8-$.1*.2^4+2*$.1*.2^3-$.1*.2^2
3 $.1^2*$.2^25+3*$.1^2*$.2^18-2*$.1^2*$.2^17+3*$.1^2*$.2^11
-4*$.1^2*$.2^10+$.1^2*$.2^9-$.1^2*$.2^8+$.1^2*$.2^4-2*$.1^2*$.2^3
+$.1^2*$.2^2
4 $.1*.2^26+3*$.1*.2^19-2*$.1*.2^18+3*$.1*.2^12-4*$.1*.2^11
+$.1*.2^10-$.1*.2^9+$.1*.2^5-2*$.1*.2^4+$.1*.2^3
5 $.2^27+3*$.2^20-2*$.2^19+3*$.2^13-4*$.2^12+$.2^11-$.2^10+$.2^6
-2*$.2^5+$.2^4
6 $.2^26+3*$.2^19-2*$.2^18+3*$.2^12-4*$.2^11+$.2^10-$.2^9+$.2^5
-2*$.2^4+$.2^3

```

char 11

time for q= 13 is 1.160 seconds
modulus= 2145

```

1 $.1^4+$.1^3*$.2^23+$.1^3*$.2^22+$.1^3*$.2^21+3*$.1^3*$.2^16
+$.1^3*$.2^15+$.1^3*$.2^14-$.1^3*$.2^13+3*$.1^3*$.2^9-$.1^3*$.2^8
-2*$.1^3*$.2^6-$.1^3*$.2^5-$.1^3*$.2^4+$.1^3*$.2^2-$.1^3*$.2
-$.1^2*$.2-$.1*.2^24-$.1*.2^23-$.1*.2^22-3*$.1*.2^17-$.1*.2^16
-$.1*.2^15+$.1*.2^14-3*$.1*.2^10+$.1*.2^9+2*$.1*.2^7
+$.1*.2^6-$.1*.2^3+$.1*.2^2-$.2^18-2*$.2^11+$.2^10-$.2^4+$.2^3
2 $.1^3*$.2^24+3*$.1^3*$.2^17-2*$.1^3*$.2^16+3*$.1^3*$.2^10
-4*$.1^3*$.2^9+$.1^3*$.2^8-$.1^3*$.2^7+$.1^3*$.2^3-2*$.1^3*$.2^2
+$.1^3*$.2-$.1*.2^25-3*$.1*.2^18+2*$.1*.2^17-3*$.1*.2^11
+4*$.1*.2^10-$.1*.2^9+$.1*.2^8-$.1*.2^4+2*$.1*.2^3-$.1*.2^2
3 $.1^2*$.2^25+3*$.1^2*$.2^18-2*$.1^2*$.2^17+3*$.1^2*$.2^11
-4*$.1^2*$.2^10+$.1^2*$.2^9-$.1^2*$.2^8+$.1^2*$.2^4-2*$.1^2*$.2^3
+$.1^2*$.2^2
4 $.1*.2^26+3*$.1*.2^19-2*$.1*.2^18+3*$.1*.2^12-4*$.1*.2^11
+$.1*.2^10-$.1*.2^9+$.1*.2^5-2*$.1*.2^4+$.1*.2^3
5 $.2^27+3*$.2^20-2*$.2^19+3*$.2^13-4*$.2^12+$.2^11-$.2^10+$.2^6
-2*$.2^5+$.2^4
6 $.2^26+3*$.2^19-2*$.2^18+3*$.2^12-4*$.2^11+$.2^10-$.2^9+$.2^5
-2*$.2^4+$.2^3

```

char 13

GQ [
\$.1^2-\$.1*.5^29-2*\$.1*.5^28-3*\$.1*.5^27+\$.2*.5^28-2*\$.1*.5^26
+3*\$.2*.5^27-\$.1*.5^25+6*\$.2*.5^26+7*\$.2*.5^25+6*\$.2*.5^24
-4*\$.1*.5^22+3*\$.2*.5^23-5*\$.1*.5^21+\$.2*.5^22-6*\$.1*.5^20

$$\begin{aligned}
&+4*$.2*$.5^{21}-$.1*$.5^{19}+9*$.2*$.5^{20}+15*$.2*$.5^{19}+$.1*$.5^{17} \\
&+13*$.2*$.5^{18}+9*$.2*$.5^{17}-6*$.1*$.5^{15}+3*$.2*$.5^{16} \\
&-3*$.1*$.5^{14}+$.2*$.5^{15}-3*$.1*$.5^{13}+6*$.2*$.5^{14}+4*$.1*$.5^{12} \\
&+9*$.2*$.5^{13}+3*$.1*$.5^{11}+12*$.2*$.5^{12}+4*$.1*$.5^{10} \\
&+5*$.2*$.5^{11}+2*$.1*$.5^9-3*$.1*$.5^8-6*$.2*$.5^9+$.1*$.5^7 \\
&-7*$.2*$.5^8-2*$.2*$.5^7+3*$.1*$.5^5+$.1*$.5^4+2*$.2*$.5^5 \\
&+$.1*$.5^3-$.2*$.5^4-$.1*$.5^2-2*$.2*$.5^3-2*$.1*$.5 \\
&-3*$.2*$.5^2-2*$.2*$.5-$.5^{21}-2*$.5^{20}-3*$.5^{19}-2*$.5^{18} \\
&-.5^{17}-3*$.5^{14}-4*$.5^{13}-5*$.5^{12}-2*$.5^{11}-$.5^{10}-3*$.5^7 \\
&-2*$.5^6-2*$.5^5-1,
\end{aligned}$$

$$\begin{aligned}
&$.1*$.2-$.1*$.5^{30}-$.1*$.5^{29}-$.1*$.5^{28}+$.2*$.5^{29}+2*$.2*$.5^{28} \\
&+3*$.2*$.5^{27}+2*$.2*$.5^{26}+$.2*$.5^{25}-4*$.1*$.5^{23}-$.1*$.5^{22} \\
&-.1*$.5^{21}+4*$.2*$.5^{22}+2*$.1*$.5^{20}+5*$.2*$.5^{21}+6*$.2*$.5^{20} \\
&+$.2*$.5^{19}-6*$.1*$.5^{16}-$.2*$.5^{17}+3*$.1*$.5^{15}+6*$.2*$.5^{15} \\
&+4*$.1*$.5^{13}+3*$.2*$.5^{14}+3*$.2*$.5^{13}+$.1*$.5^{11}-4*$.2*$.5^{12} \\
&-3*$.2*$.5^{11}-4*$.1*$.5^9-4*$.2*$.5^{10}+5*$.1*$.5^8-2*$.2*$.5^9 \\
&-.1*$.5^7+3*$.2*$.5^8+2*$.1*$.5^6-$.2*$.5^7-$.1*$.5^5 \\
&-3*$.2*$.5^5-$.1*$.5^3-$.2*$.5^4-$.1*$.5^2-$.2*$.5^3+2*$.1*$.5 \\
&-.1+$.2*$.5-$.2-$.5^{22}-$.5^{21}-$.5^{20}-3*$.5^{15}-$.5^{14}-$.5^{13}+$.5^{12} \\
&-3*$.5^8+$.5^7+$.5^5-$.5+1,
\end{aligned}$$

$$\begin{aligned}
&$.2^2-$.1*$.5^{31}+$.2*$.5^{30}+$.2*$.5^{29}+$.2*$.5^{28}-4*$.1*$.5^{24} \\
&+3*$.1*$.5^{23}+4*$.2*$.5^{23}+$.2*$.5^{22}+$.2*$.5^{21}-2*$.2*$.5^{20} \\
&-6*$.1*$.5^{17}+9*$.1*$.5^{16}-3*$.1*$.5^{15}+6*$.2*$.5^{16}+$.1*$.5^{14} \\
&-3*$.2*$.5^{15}-4*$.2*$.5^{13}-4*$.1*$.5^{10}-$.2*$.5^{11}+9*$.1*$.5^9 \\
&-6*$.1*$.5^8+4*$.2*$.5^9+2*$.1*$.5^7-5*$.2*$.5^8-$.1*$.5^6 \\
&+$.2*$.5^7-2*$.2*$.5^6+$.2*$.5^5-$.1*$.5^3+3*$.1*$.5^2 \\
&-3*$.1*$.5+$.2*$.5^2+$.1-2*$.2*$.5+$.2-$.5^{23}-3*$.5^{16} \\
&+2*$.5^{15}-3*$.5^9+4*$.5^8-$.5^7-$.5^2+2*$.5-1,
\end{aligned}$$

$$\begin{aligned}
&$.1*$.3-$.1*$.5^8-$.1*$.5-$.3*$.5^2-$.3*$.5-$.3+$.5^{10}+$.5^9 \\
&+$.5^8+$.5^3+$.5^2+$.5,
\end{aligned}$$

$$$.2*$.3-$.2*$.5^8-$.2*$.5-$.3*$.5^3+$.5^{11}+$.5^4,$$

$$\begin{aligned}
&$.1*$.4-$.1*$.5^{23}-$.1*$.5^{22}-$.1*$.5^{21}+$.2*$.5^{22}+2*$.2*$.5^{21} \\
&+3*$.2*$.5^{20}+2*$.2*$.5^{19}+$.2*$.5^{18}-3*$.1*$.5^{16}-$.1*$.5^{15} \\
&-.1*$.5^{14}+3*$.2*$.5^{15}+$.1*$.5^{13}+4*$.2*$.5^{14}+5*$.2*$.5^{13} \\
&+2*$.2*$.5^{12}+$.2*$.5^{11}-3*$.1*$.5^9+$.1*$.5^8+3*$.2*$.5^8 \\
&+2*$.1*$.5^6+2*$.2*$.5^7+$.1*$.5^5+2*$.2*$.5^6+$.1*$.5^4 \\
&-$.2*$.5^5-2*$.2*$.5^4-$.1*$.5^2-3*$.2*$.5^3+$.1*$.5 \\
&-2*$.2*$.5^2-$.4*$.5^2-$.4*$.5-$.4-$.5^{15}-$.5^{14}-$.5^{13} \\
&-2*$.5^8-$.5^7-$.5^6-$.5,
\end{aligned}$$

$$$.3^2-$.1*$.5^{24}+$.2*$.5^{23}+$.2*$.5^{22}+$.2*$.5^{21}-3*$.1*$.5^{17}$$

$$\begin{aligned}
&+2*\$.1*\$.5^{16}+3*$.2*\$.5^{16}+$.2*\$.5^{15}+$.2*\$.5^{14}-$.2*\$.5^{13} \\
&-3*$.1*\$.5^{10}+4*$.1*\$.5^9-$.1*\$.5^8+3*$.2*\$.5^9+$.1*\$.5^7 \\
&-\$.2*\$.5^8-2*$.2*\$.5^6-$.2*\$.5^5-$.1*\$.5^3-$.2*\$.5^4 \\
&+2*$.1*\$.5^2-$.1*\$.5+$.2*\$.5^2-$.2*\$.5-$.3-$.4*$.5^3-$.5^{16} \\
&-2*$.5^9+$.5^8-$.5^2+$.5,
\end{aligned}$$

$$\begin{aligned}
&$.2*\$.4-$.1*\$.5^{24}+$.2*\$.5^{23}+$.2*\$.5^{22}+$.2*\$.5^{21}-3*$.1*\$.5^{17} \\
&+2*$.1*\$.5^{16}+3*$.2*\$.5^{16}+$.2*\$.5^{15}+$.2*\$.5^{14}-$.2*\$.5^{13} \\
&-3*$.1*\$.5^{10}+4*$.1*\$.5^9-$.1*\$.5^8+3*$.2*\$.5^9+$.1*\$.5^7 \\
&-\$.2*\$.5^8-2*$.2*\$.5^6-$.2*\$.5^5-$.1*\$.5^3-$.2*\$.5^4 \\
&+2*$.1*\$.5^2-$.1*\$.5+$.2*\$.5^2-$.2*\$.5-$.4*$.5^3-$.5^{16} \\
&-2*$.5^9+$.5^8-$.5^2+$.5,
\end{aligned}$$

$$$.3*\$.4-$.2*\$.5-$.4,$$

$$$.4^2-$.3*\$.5$$

]
WT= [
[74, 73, 49, 25, 1]
]

PHI:

- 1 $$.1^4+$.1^3*$.2^{23}+$.1^3*$.2^{22}+$.1^3*$.2^{21}+3*$.1^3*$.2^{16}$
 $+$.1^3*$.2^{15}+$.1^3*$.2^{14}-$.1^3*$.2^{13}+3*$.1^3*$.2^9-$.1^3*$.2^8$
 $-2*$.1^3*$.2^6-$.1^3*$.2^5-$.1^3*$.2^4+$.1^3*$.2^2-$.1^3*$.2$
 $-\$.1^2*$.2-$.1*$.2^4-$.1*$.2^3-$.1*$.2^2-3*$.1*$.2^{17}-$.1*$.2^{16}$
 $-\$.1*$.2^{15}+$.1*$.2^{14}-3*$.1*$.2^{10}+$.1*$.2^9+2*$.1*$.2^7$
 $+$.1*$.2^6-$.1*$.2^3+$.1*$.2^2-$.2^{18}-2*$.2^{11}+$.2^{10}-$.2^4+$.2^3$
- 2 $$.1^3*$.2^{24}+3*$.1^3*$.2^{17}-2*$.1^3*$.2^{16}+3*$.1^3*$.2^{10}$
 $-4*$.1^3*$.2^9+$.1^3*$.2^8-$.1^3*$.2^7+$.1^3*$.2^3-2*$.1^3*$.2^2$
 $+$.1^3*$.2-$.1*$.2^5-3*$.1*$.2^{18}+2*$.1*$.2^{17}-3*$.1*$.2^{11}$
 $+4*$.1*$.2^{10}-$.1*$.2^9+$.1*$.2^8-$.1*$.2^4+2*$.1*$.2^3-$.1*$.2^2$
- 3 $$.1^2*$.2^{25}+3*$.1^2*$.2^{18}-2*$.1^2*$.2^{17}+3*$.1^2*$.2^{11}$
 $-4*$.1^2*$.2^{10}+$.1^2*$.2^9-$.1^2*$.2^8+$.1^2*$.2^4-2*$.1^2*$.2^3$
 $+$.1^2*$.2^2$
- 4 $$.1*$.2^{26}+3*$.1*$.2^{19}-2*$.1*$.2^{18}+3*$.1*$.2^{12}-4*$.1*$.2^{11}$
 $+$.1*$.2^{10}-$.1*$.2^9+$.1*$.2^5-2*$.1*$.2^4+$.1*$.2^3$
- 5 $$.2^{27}+3*$.2^{20}-2*$.2^{19}+3*$.2^{13}-4*$.2^{12}+$.2^{11}-$.2^{10}+$.2^6$
 $-2*$.2^5+$.2^4$
- 6 $$.2^{26}+3*$.2^{19}-2*$.2^{18}+3*$.2^{12}-4*$.2^{11}+$.2^{10}-$.2^9+$.2^5-2*$.2^4+$.2^3$

PSIQ:

- 1 \$.4
- 2 \$.5

IQ:

- 1 $0.4^2 - 0.3 * 0.5$
- 2 $0.3^2 - 0.1 * 0.5^{24} + 0.2 * 0.5^{23} + 0.2 * 0.5^{22} + 0.2 * 0.5^{21} - 3 * 0.1 * 0.5^{17}$
 $+ 2 * 0.1 * 0.5^{16} + 3 * 0.2 * 0.5^{16} + 0.2 * 0.5^{15} + 0.2 * 0.5^{14} - 0.2 * 0.5^{13}$
 $- 3 * 0.1 * 0.5^{10} + 4 * 0.1 * 0.5^9 - 0.1 * 0.5^8 + 3 * 0.2 * 0.5^9 + 0.1 * 0.5^7 - 0.2 * 0.5^8$
 $- 2 * 0.2 * 0.5^6 - 0.2 * 0.5^5 - 0.1 * 0.5^3 - 0.2 * 0.5^4 + 2 * 0.1 * 0.5^2 - 0.1 * 0.5$
 $+ 0.2 * 0.5^2 - 0.2 * 0.5 - 0.3 - 0.4 * 0.5^3 - 0.5^{16} - 2 * 0.5^9 + 0.5^8 - 0.5^2 + 0.5$
- 3 $0.3 * 0.4 - 0.2 * 0.5 - 0.4$
- 4 $0.2^2 - 0.1 * 0.5^{31} + 0.2 * 0.5^{30} + 0.2 * 0.5^{29} + 0.2 * 0.5^{28} - 4 * 0.1 * 0.5^{24}$
 $+ 3 * 0.1 * 0.5^{23} + 4 * 0.2 * 0.5^{23} + 0.2 * 0.5^{22} + 0.2 * 0.5^{21} - 2 * 0.2 * 0.5^{20}$
 $- 6 * 0.1 * 0.5^{17} + 9 * 0.1 * 0.5^{16} - 3 * 0.1 * 0.5^{15} + 6 * 0.2 * 0.5^{16} + 0.1 * 0.5^{14}$
 $- 3 * 0.2 * 0.5^{15} - 4 * 0.2 * 0.5^{13} - 4 * 0.1 * 0.5^{10} - 0.2 * 0.5^{11} + 9 * 0.1 * 0.5^9$
 $- 6 * 0.1 * 0.5^8 + 4 * 0.2 * 0.5^9 + 2 * 0.1 * 0.5^7 - 5 * 0.2 * 0.5^8 - 0.1 * 0.5^6$
 $+ 0.2 * 0.5^7 - 2 * 0.2 * 0.5^6 + 0.2 * 0.5^5 - 0.1 * 0.5^3 + 3 * 0.1 * 0.5^2 - 3 * 0.1 * 0.5$
 $+ 0.2 * 0.5^2 + 0.1 - 2 * 0.2 * 0.5 + 0.2 - 0.5^{23} - 3 * 0.5^{16} + 2 * 0.5^{15} - 3 * 0.5^9$
 $+ 4 * 0.5^8 - 0.5^7 - 0.5^2 + 2 * 0.5 - 1$
- 5 $0.2 * 0.3 - 0.2 * 0.5^8 - 0.2 * 0.5 - 0.3 * 0.5^3 + 0.5^{11} + 0.5^4$
- 6 $0.2 * 0.4 - 0.1 * 0.5^{24} + 0.2 * 0.5^{23} + 0.2 * 0.5^{22} + 0.2 * 0.5^{21} - 3 * 0.1 * 0.5^{17}$
 $+ 2 * 0.1 * 0.5^{16} + 3 * 0.2 * 0.5^{16} + 0.2 * 0.5^{15} + 0.2 * 0.5^{14} - 0.2 * 0.5^{13}$
 $- 3 * 0.1 * 0.5^{10} + 4 * 0.1 * 0.5^9 - 0.1 * 0.5^8 + 3 * 0.2 * 0.5^9 + 0.1 * 0.5^7 - 0.2 * 0.5^8$
 $- 2 * 0.2 * 0.5^6 - 0.2 * 0.5^5 - 0.1 * 0.5^3 - 0.2 * 0.5^4 + 2 * 0.1 * 0.5^2 - 0.1 * 0.5$
 $+ 0.2 * 0.5^2 - 0.2 * 0.5 - 0.4 * 0.5^3 - 0.5^{16} - 2 * 0.5^9 + 0.5^8 - 0.5^2 + 0.5$
- 7 $0.1^2 - 0.1 * 0.5^{29} - 2 * 0.1 * 0.5^{28} - 3 * 0.1 * 0.5^{27} + 0.2 * 0.5^{28} - 2 * 0.1 * 0.5^{26}$
 $+ 3 * 0.2 * 0.5^{27} - 0.1 * 0.5^{25} + 6 * 0.2 * 0.5^{26} + 7 * 0.2 * 0.5^{25} + 6 * 0.2 * 0.5^{24}$
 $- 4 * 0.1 * 0.5^{22} + 3 * 0.2 * 0.5^{23} - 5 * 0.1 * 0.5^{21} + 0.2 * 0.5^{22} - 6 * 0.1 * 0.5^{20}$
 $+ 4 * 0.2 * 0.5^{21} - 0.1 * 0.5^{19} + 9 * 0.2 * 0.5^{20} + 15 * 0.2 * 0.5^{19} + 0.1 * 0.5^{17}$
 $+ 13 * 0.2 * 0.5^{18} + 9 * 0.2 * 0.5^{17} - 6 * 0.1 * 0.5^{15} + 3 * 0.2 * 0.5^{16} - 3 * 0.1 * 0.5^{14}$
 $+ 0.2 * 0.5^{15} - 3 * 0.1 * 0.5^{13} + 6 * 0.2 * 0.5^{14} + 4 * 0.1 * 0.5^{12} + 9 * 0.2 * 0.5^{13}$
 $+ 3 * 0.1 * 0.5^{11} + 12 * 0.2 * 0.5^{12} + 4 * 0.1 * 0.5^{10} + 5 * 0.2 * 0.5^{11} + 2 * 0.1 * 0.5^9$
 $- 3 * 0.1 * 0.5^8 - 6 * 0.2 * 0.5^9 + 0.1 * 0.5^7 - 7 * 0.2 * 0.5^8 - 2 * 0.2 * 0.5^7$
 $+ 3 * 0.1 * 0.5^5 + 0.1 * 0.5^4 + 2 * 0.2 * 0.5^5 + 0.1 * 0.5^3 - 0.2 * 0.5^4 - 0.1 * 0.5^2$
 $- 2 * 0.2 * 0.5^3 - 2 * 0.1 * 0.5 - 3 * 0.2 * 0.5^2 - 2 * 0.2 * 0.5 - 0.5^{21} - 2 * 0.5^{20}$
 $- 3 * 0.5^{19} - 2 * 0.5^{18} - 0.5^{17} - 3 * 0.5^{14} - 4 * 0.5^{13} - 5 * 0.5^{12} - 2 * 0.5^{11}$
 $- 0.5^{10} - 3 * 0.5^7 - 2 * 0.5^6 - 2 * 0.5^5 - 1$
- 8 $0.1 * 0.2 - 0.1 * 0.5^{30} - 0.1 * 0.5^{29} - 0.1 * 0.5^{28} + 0.2 * 0.5^{29} + 2 * 0.2 * 0.5^{28}$
 $+ 3 * 0.2 * 0.5^{27} + 2 * 0.2 * 0.5^{26} + 0.2 * 0.5^{25} - 4 * 0.1 * 0.5^{23} - 0.1 * 0.5^{22}$
 $- 0.1 * 0.5^{21} + 4 * 0.2 * 0.5^{22} + 2 * 0.1 * 0.5^{20} + 5 * 0.2 * 0.5^{21} + 6 * 0.2 * 0.5^{20}$
 $+ 0.2 * 0.5^{19} - 6 * 0.1 * 0.5^{16} - 0.2 * 0.5^{17} + 3 * 0.1 * 0.5^{15} + 6 * 0.2 * 0.5^{15}$
 $+ 4 * 0.1 * 0.5^{13} + 3 * 0.2 * 0.5^{14} + 3 * 0.2 * 0.5^{13} + 0.1 * 0.5^{11} - 4 * 0.2 * 0.5^{12}$
 $- 3 * 0.2 * 0.5^{11} - 4 * 0.1 * 0.5^9 - 4 * 0.2 * 0.5^{10} + 5 * 0.1 * 0.5^8 - 2 * 0.2 * 0.5^9$
 $- 0.1 * 0.5^7 + 3 * 0.2 * 0.5^8 + 2 * 0.1 * 0.5^6 - 0.2 * 0.5^7 - 0.1 * 0.5^5 - 3 * 0.2 * 0.5^5$
 $- 0.1 * 0.5^3 - 0.2 * 0.5^4 - 0.1 * 0.5^2 - 0.2 * 0.5^3 + 2 * 0.1 * 0.5 - 0.1 * 0.2 * 0.5$
 $- 0.2 - 0.5^{22} - 0.5^{21} - 0.5^{20} - 3 * 0.5^{15} - 0.5^{14} - 0.5^{13} + 0.5^{12}$
 $- 3 * 0.5^8 + 0.5^7 + 0.5^5 - 0.5 + 1$
- 9 $0.1 * 0.3 - 0.1 * 0.5^8 - 0.1 * 0.5 - 0.3 * 0.5^2 - 0.3 * 0.5 - 0.3 + 0.5^{10} + 0.5^9$
 $+ 0.5^8 + 0.5^3 + 0.5^2 + 0.5$
- 10 $0.1 * 0.4 - 0.1 * 0.5^{23} - 0.1 * 0.5^{22} - 0.1 * 0.5^{21} + 0.2 * 0.5^{22} + 2 * 0.2 * 0.5^{21}$

+3*\$.2*\$.5^20+2*\$.2*\$.5^19+\$.2*\$.5^18-3*\$.1*\$.5^16-\$.1*\$.5^15
-\$.1*\$.5^14+3*\$.2*\$.5^15+\$.1*\$.5^13+4*\$.2*\$.5^14+5*\$.2*\$.5^13
+2*\$.2*\$.5^12+\$.2*\$.5^11-3*\$.1*\$.5^9+\$.1*\$.5^8+3*\$.2*\$.5^8
+2*\$.1*\$.5^6+2*\$.2*\$.5^7+\$.1*\$.5^5+2*\$.2*\$.5^6+\$.1*\$.5^4-\$.2*\$.5^5
-2*\$.2*\$.5^4-\$.1*\$.5^2-3*\$.2*\$.5^3+\$.1*\$.5-2*\$.2*\$.5^2-\$.4*\$.5^2
-\$.4*\$.5-\$.4-\$.5^15-\$.5^14-\$.5^13-2*\$.5^8-\$.5^7-\$.5^6-\$.5

totaltime= 34.650 seconds
34.650

Loading "intclos_min061809.mag"
q= 0
